ADVANCED CALCULUS MAA4102
FIRST HOUR EXAM
FALL 2006

Name:

No calculators permitted during the exam.
Each problem is worth 20 points.
Explain all answers!

1.

a. Give a careful definition of what it means for a sequence to converge to a number L.

b. Using the DEFINITION of limit show that $lim_{n\to\infty}\frac{7n+2}{5n+3}=\frac{7}{5}$.
(You will receive zero credit for using any limit theorems.)

c. Using limit theorems compute $lim_{n\to\infty}(\sqrt{4n+7}-\sqrt{4n})$.

2.

a. State the square root algorithm of Archimedes/Heron.

b. Use the square root algorithm of Archimedes/Heron to compute three approximations of $\sqrt{11}$. (i.e. If $x_0 = 1$, then compute x_1, x_2, and x_3.)

c. If $K > 0$ and x_n denotes the n^{th} term in the Archimedes/Heron algorithm to approximate $\sqrt{K}$ and $x_0 = 1$, then show that $x_{n+1} \leq x_n$.

3.

a. Give a careful statement of the least upper bound principle.

b. Determine the least upper bound of the set $S = \{x \in \Re : x^3 < 5\}$.

4.

a. Prove: If $\lim_{n\to\infty} x_n = L$ and $\lim_{n\to\infty} y_n = M$, then $\lim_{n\to\infty}(x_n + y_n) = L + M$.

b. Prove: If a sequence $\{x_n\}_{n=1}^{\infty}$ converges to a number L, then it is bounded.

5.

a. Give a careful definition of what it means for a sequence to be *Cauchy*.

b. Determine whether or not the sequence $x_n = (-1)^n \frac{n+1}{n}$ is Cauchy. Explain your answer.