ADVANCED CALCULUS MAA4102
SECOND HOUR EXAM
FALL 2005

Name:

No calculators permitted during the exam.
Each problem is worth 20 points.
Explain all answers!

1.

a. Give a careful definition of what it means for a sequence to be Cauchy.

b. Prove: If a sequence is Cauchy, then it converges.

2.

a. Show the series $\sum_{n=1}^{\infty} \frac{1}{n^n}$ converges. (Be sure to explain your answer.)

b. Find an integer N with the property that if $n \geq N$, then $|\sum_{n=1}^{n} \frac{1}{n^n} - \sum_{n=1}^{N} \frac{1}{n^n}| < \frac{1}{10^3}$. Justify your answer. (Think geometric.)

3.

a. Give a careful statement of the Mean Value Theorem.

b. Prove the Mean Value Theorem.

4.

a. Give a careful statement of the Intermediate Value Theorem.

b. Prove: If $f(x) : \Re \to \Re$ is defined by $f(x) = x^5 + 5x + 1$, then there is at least one point $z \in \Re$ with the property that $f(z) = 47$.

c. Show that $f(x) = x^5 + 5x + 1$ has the property that there is exactly one point $z \in \Re$ such that $f(z) = 47$.

5.

a. Prove: If n is a positive integer, then the function $f(x) = (1+x)^{\frac{1}{n}} - (x)^{\frac{1}{n}}$ is decreasing for all $x \in [0, \infty)$.

b. If $f(x) : \Re \to \Re$ is differentiable at every $x \in \Re$ and $f(x)$ is strictly increasing, then is it necessarily true that $f'(x) > 0$ for all $x \in \Re$? Explain!