ADVANCED CALCULUS MAA4102
SECOND HOUR EXAM
FALL 2006

Name:

No calculators permitted during the exam.
Each problem is worth 20 points.
Explain all answers!

1.

a. Give a careful definition of $\lim_{x \to x_0} f(x) = L$.

b. Using the DEFINITION, show that $\lim_{x \to 0} \frac{(x+1)^2 - 1}{x} = 2$.

2.

a. Determine whether or not the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ converges or diverges. (Explain your answer!)

b. Show the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges.

c. Find an integer N with the property that $|\sum_{n=N}^{\infty} \frac{(-1)^n}{\sqrt{n}}| < \frac{1}{10^3}$. (Justify your answer.)

3.

a. Give a careful statement of the Intermediate Value Theorem for Integrals II.

b. Prove the Intermediate Value Theorem for Integrals II.

4.

a. Give a careful statement of BOTH parts of the Fundamental Theorem of Calculus.

b. Prove the first part of the Fundamental Theorem of Calculus.

c. Explain the relationship between the Fundamental Theorem of Calculus and the volume and surface area formulas for a sphere.

5.

a. Give a careful statement of Taylor's Theorem.

b. If $f(x) = \sin(\frac{2}{3}x)$, then compute the 5^{th} degree Taylor polynomial at $x_0 = 0$ which approximates $f(x)$.

c. If $f(x) = \sin(\frac{2}{3}x), x_0 = 0$, and $tol = \frac{1}{10^6}$, then find an integer n with the property that difference between $f(x)$ and the degree n Taylor series approximation at $x_0 = 0$ is less than $tol = \frac{1}{10^6}$ for all $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.
(Simply set up the requisite inequality which determines n.)