ADVANCED CALCULUS MAA4102
SECOND HOUR EXAM
SPRING 2004

Name:

No calculators permitted during the exam.
Each problem is worth 25 points.
Explain all answers!

1.

a. Give a careful statement of the mean value theorem.

b. Prove the mean value theorem.

c. If $f(x)$ is a continuous function on an interval $[a, b]$, then prove that any two antiderivatives of $f(x)$ differ by a constant.

2.

a. Give a careful statement of the intermediate value theorem.

b. If $f(x) = x^3 + 3x^2 + 10x + 5$, then show that $f(x)$ has at least one real root.

c. If $f(x) = x^3 + 3x^2 + 10x + 5$, then show that $f(x)$ cannot have two distinct real roots.

d. Indicate how the bisection method can be used to approximate the real root of $f(x) = x^3 + 3x^2 + 10x + 5$.

3.

a. Evaluate $\lim_{x \to 0} \frac{1-\cos(x)}{x^2}$.

b. Show that the function $f(x) = x^2 + 3x + 1$ is uniformly continuous on the interval $[-2, 2]$.

c. Show that the function $f(x) = x^3$ is NOT uniformly continuous on the interval $(-\infty, \infty)$.

4.

Prove: The nested intersection of closed bounded intervals is non-empty.