

ADVANCED CALCULUS MAA4102
FIRST HOUR EXAM
SPRING 2004

Name:

No calculators permitted during the exam.

Each problem is worth 25 points.

Explain all answers!

1.

a. State the square root algorithm of Archimedes/Heron.

b. Use the algorithm of Archimedes/Heron to approximate the square root of 17. (Compute the first three steps in the method.)

2.

a. Give a careful statement of what it means for a sequence to converge to a number L .

b. Using the DEFINITION of limit show that $\lim_{n \rightarrow \infty} \frac{1}{n^5} = 0$.

c. Using the DEFINITION of limit show that $\lim_{n \rightarrow \infty} \frac{2n-1}{3n+1} = \frac{2}{3}$.

3.

a. Determine the least upper bound (or sup) of the set $S = \{x : 2 + x > \frac{2}{2-x}\}$.

b. Determine whether or not the sequence $\{(1 - \frac{1}{2^2})(1 - \frac{1}{3^2}) \dots (1 - \frac{1}{n^2})\}_{n=2}^{\infty}$ converges. If it converges, determine the limit.

4.

a. Prove the sum formula for the geometric series.

b. Give a careful statement of the least upper bound principle.

c. Prove that every bounded increasing sequence converges.