ADVANCED CALCULUS MAA4102
THIRD HOUR EXAM
FALL 2004

Name:

No calculators permitted during the exam.
Each problem is worth 20 points.
Explain all answers!

1.

a. Give a careful statement of the Intermediate Value Theorem.

b. Prove the Intermediate Value Theorem.

2.

a. Give a careful statement of Taylor's Theorem. (Be sure to include the error term.)

b. If $f(x) = \sin(3x)$ for $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and $tol = \frac{1}{10^5}$, then find an integer n so that the n^{th} degree Taylor polynomial approximates $f(x)$ with error less than $\frac{1}{10^5}$.

3.

a. Give a careful statement of the theorem for polynomial interpolation. (Be sure to include the error term.)

b. Given the three data points $(1, 5), 3, -1), (8, 6)$, find the 2^{nd} degree polynomial which interpolates the data. (Set up the formulas or equations, but PLEASE do not simplify your answer.)

c. If $f(x) = \sin(3x)$ for $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and $tol = \frac{1}{10^5}$, and a partition $P = \{-\frac{\pi}{2} = x_0 < x_1 < x_2 < \cdots < x_n = \frac{\pi}{2}\}$ with equally spaced points, then find an integer n so that the n^{th} degree interpolating polynomial approximates $f(x)$ with error less than tol.

4.

a. If $f(x) = \sum_{n=0}^{\infty}(2n)^2 x^n$, then determine the interval of convergence for $f(x)$.

b. If $f(x) = \sum_{n=0}^{\infty}(2n)^2 x^n$, then write $f(x)$ in closed form. (i.e. Write $f(x)$ as the quotient of two polynomials.)

5.

a. Define the function $\log_e(x)$ for $x > 0$.

b. Prove: If $x, y > 0$, then $\log_e(xy) = \log_e(x) + \log_e(y)$.

c. Explain how a slide rule can be used to compute $3 * 2 = 6$.