ADVANCED CALCULUS MAA4102
THIRD HOUR EXAM
FALL 2005

Name:

No calculators permitted during the exam.
Each problem is worth 20 points.
Explain all answers!

1.

a. Give a careful statement of both parts of the Fundamental Theorem of Calculus.

b. Prove both parts of the Fundamental Theorem of Calculus. (Explain each step)

2.

a. Give a careful statement of Taylor's Theorem. (Be sure to include the error term.)

b. If $f(x) = e^{2x}$ for $x \in [-3, 3]$ and $tol = \frac{1}{10^5}$, then find an integer n so that the n^{th} degree Taylor polynomial approximates $f(x)$ with error less than $\frac{1}{10^5}$.

3.

a. Give a careful statement of the intermediate value theorem for integrals.

b. Prove: the intermediate value theorem for integrals.

4.

a. Explain why the following argument is incorrect. $\int_{-1}^{1} \frac{1}{x^2}\, dx = -\frac{1}{x}|_{-1}^{1} = -1 - 1 = -2$.

b. Prove: If $x \geq 1$, then $\ln(x) = log_e(x) \leq x - 1$.

5.

a. Give a careful definition of what it means for the integral of a function $f(x) : [a, b] \to \Re$ to exists.

b. Using a DEFINITION of the integral, prove: If $a \leq c \leq d \leq b, f(x) : [a, b] \to \Re$, and $\int_a^b f(x)\, dx$ exists, then $\int_c^d f(x)\, dx$ exists.