ADVANCED CALCULUS MAA4102
THIRD HOUR EXAM
FALL 2006

Name:

No calculators permitted during the exam.
Each problem is worth 20 points.
Explain all answers!
1.

a. Give a careful statement of the Mean Value Theorem.

b. Prove: If $f(x) : [a, b] \to \Re$ is differentiable and has the property that $f'(x)$ is continuous at each $x \in [a, b]$, then there is a constant M with the property that $|f(x) - f(y)| \leq M|x - y|$ for all $x, y \in [a, b]$.

c. Prove: $|\sin(x) - \sin(y)| \leq |x - y|$ for all $x, y \in \Re$.

2.

a. Give a careful statement of Taylor's Theorem. (Be sure to include the error term.)

b. If $f(x) = \sin(\frac{1}{7}x)$ for $x \in [-\pi, \pi]$ and $tol = \frac{1}{10^6}$, then find an integer n so that the n^{th} degree Taylor polynomial approximates $f(x)$ with error less than $\frac{1}{10^6}$ for all $x \in [-\pi, \pi]$.

3.

a. If $f(x) : [a,b] \to \Re$ is a function, then give a careful definition of the integral $\int_a^b f(x)\ dx$.

b. If $F(x) = \int_{x^2}^{\cos(x)} \tan(x+1)\ dx$, then compute $F'(x)$.

4.

a. Give a careful definition of the function $\ln(x) = log_e(x)$.

b. Show the function $\ln(x) = log_e(x)$ is one-to-one.

c. Show the function $\ln(x) = log_e(x)$ is concave down.

d. Prove: If $x, n > 0$, then $\log_e(x^n) = nlog_e(x)$.

5.

a. Give a careful statement of the Lagrange Theorem for polynomial interpolation.

b. If $f(x) = \sin(\frac{1}{7}x)$ for $x \in [-\pi, \pi]$ and $tol = \frac{1}{10^6}$, then find an integer n so that the n^{th} degree interpolating polynomial $p_n(x)$ for equally spaced points has the property that $|f(x) - p_n(x)| < \frac{1}{10^6}$, for all $x \in [-\pi, \pi]$.