L1 Precalc review

1. Function: rule which assigns to each element in set D exactly one element in set R.

- domain = D (allowable inputs)
- range = R (outputs)
- graph: $\{(x, y) \; : \; x$ is in D and $y = f(x) \; \}$
- zero (root, solution): c is a zero of $f(x)$ if $f(c) = 0$
- even/odd function even: $f(-x) = f(x)$, odd: $f(-x) = -f(x)$
- increasing/decreasing increasing: walk uphill from L to R, decreasing: walk downhill from L to R

2. **Transform** the graph of $y = f(x)$

- $f(x) \pm c$: shifts graph up/down c units
- $f(x + c)$: shifts graph left c units while $f(x - c)$: shifts graph right c units
- $cf(x)$ is a vertical stretch if $c > 1$ and vertical shrink if $0 < c < 1$
- $f(cx)$: horizontal shrink if $c > 1$, horizontal stretch if $0 < c < 1$
- $-f(x)$: reflect across x-axis
- $f(-x)$: reflect across y-axis

Solving equations

Factor and solve $x^{-\frac{2}{3}}(2-x)^2 - 6x^{\frac{1}{3}}(2-x) = 0$

Solve $x = \sqrt{5-x^2} - 1$

Solving inequalities:

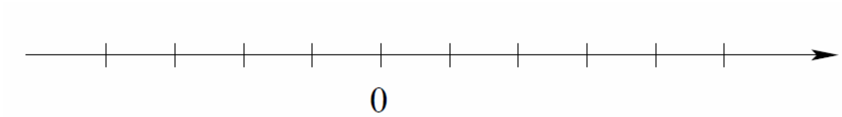
$a < b$ and $c > 0 \longrightarrow ac < bc$

$a < b$ and $c < 0 \longrightarrow ac > bc$

Find the solution set: $\frac{10-x}{x+2} \geq 2$

Absolute Value

Def. If a is a real number $|a|$



so that $|a| = \begin{cases} & a \le 0 \\ & a > 0 \end{cases}$

ex If $x \neq 0$, find an expression for $f(x) = \frac{x}{|x|}$

$$\frac{x}{|x|} = \begin{cases} \frac{x}{-x} & x < 0 \\ \frac{x}{x} & x > 0 \end{cases}$$

ex if $x \neq 1$ find an expression for

$$g(x) = \frac{2(x-1)}{|x-1|} = \begin{cases} & x < 1 \\ & x > 1 \end{cases}$$

Absolute Value Inequalities

Let $a > 0$:

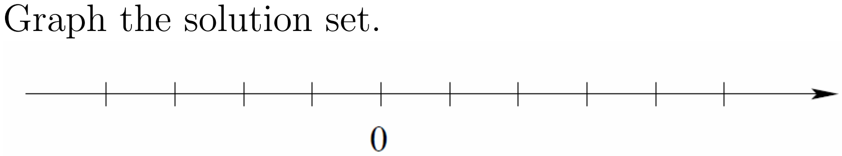
$|x| < a$ if and only if

$|x| > a$ if and only if

ex. Solve and express your answer using intervals.

$3 - |\frac{1-3x}{2}| < -1$

Graph the solution set.



Elementary functions

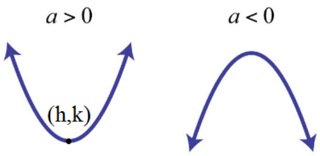
1. Polynomials $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ where n is a nonnegative integer

- linear functions $f(x) = mx + b$

  point-slope form: $y - y_1 = m(x - x_1)$ uses point $(x_1,\ y_{1)}$ and slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

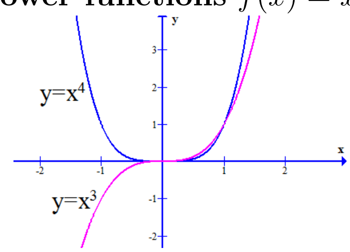
  slope-intercept form: $y = mx + b$, m =slope, $(0, b) = y$-intercept

- quadratic functions $f(x) = ax^2 + bx + c$, $a \neq 0$



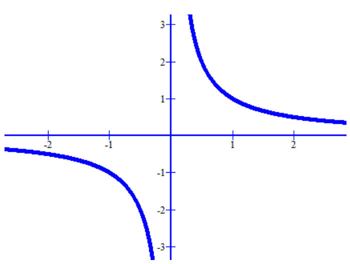
  $f(x) = a(x-h)^2 + k$ (complete the square to put in vertex form)

- **power functions** $f(x) = x^n$, n a positive integer

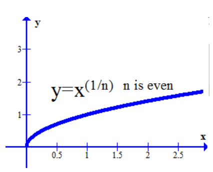


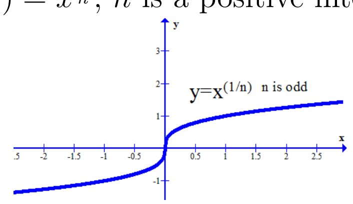
2. Rational functions $f(x) = \frac{P(x)}{Q(x)}$ where P and Q are polynomials

- reciprocal function $f(x) = \frac{1}{x}$



3. root functions $f(x) = x^{\frac{1}{n}}$, n is a positive integer





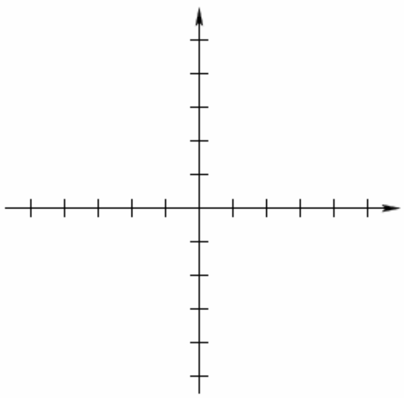
4. Algebraic functions: functions that can be constructed from polynomial functions using operations addition, subtraction, multiplcation, division, and taking roots

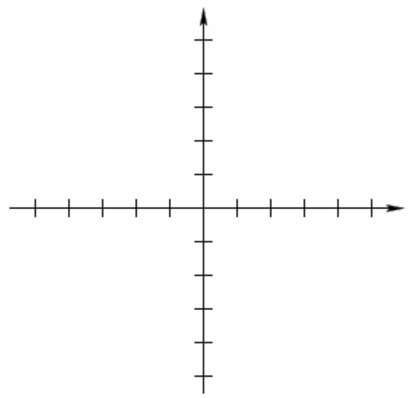
5.Transcendental functions, which are not algebraic (trigonometric functions, inverse trig functions, exponential and logarithmic functions)

Domain

$f(x) = 2x + 1$ $\qquad\qquad$ $g(x) = \frac{2x^2-3x-2}{x-2}$

Are these equivalent functions?

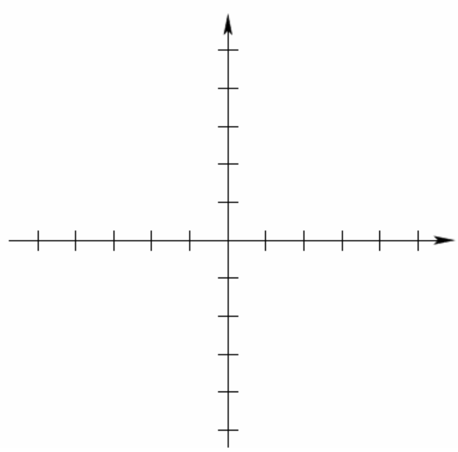




Piecewise Defined Functions

<u>**ex.**</u> a) Sketch the graph:

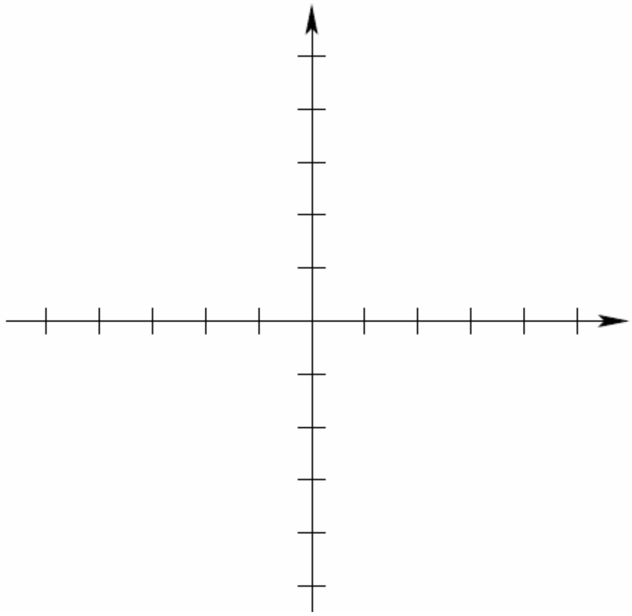
$$f(x) = \begin{cases} \dfrac{x}{|x|} & x < 0 \\ 3 - x & 0 < x < 2 \\ 2x^2 - 8x + 9 & x \geq 2 \end{cases}$$



b) Find each interval on which $f(x)$ is increasing, decreasing and constant.

Translations and Transformations

ex. $y = 2 - \sqrt{x-1}$



Use the following order to graph a function involving more than one transformation:

1. Horizontal Translation
2. Stretching or shrinking
3. Reflecting
4. Vertical Translation (done last)

Function Composition

Def. $(f \circ g)(x) =$

Def. $(g \circ f)(x) =$

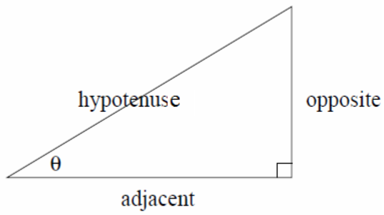
ex. If $f(x) = \dfrac{1}{x+2}$ and $g(x) = \dfrac{4}{x-1}$, find with domain:

$(f \circ g)(x)$

NOTE: The domain of $F(x) = (f \circ g)(x)$ is the intersection of the domain of inner function g and the resulting function F.

Trigonometry

1. Unit conversion: degrees $\longleftrightarrow$ radians
2. Trigonometric functions



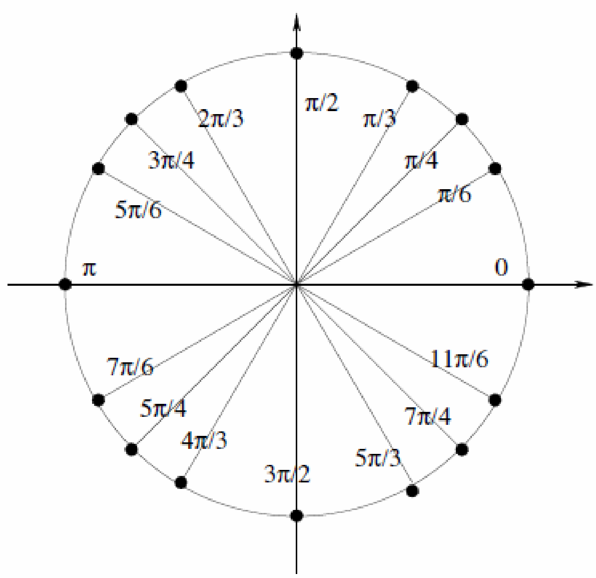
$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

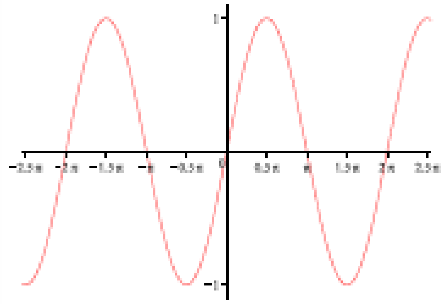
3. Two basic triangles

4. Unit circle ($r = 1$, so $\sin\theta = y$ and $\cos\theta = x$)

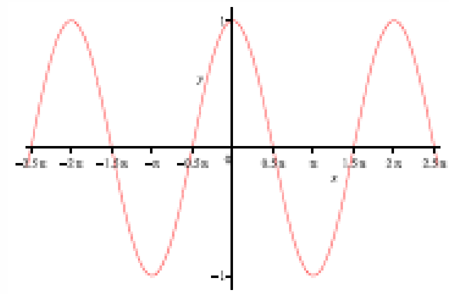


θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$					
$\cos\theta$					
$\tan\theta$					

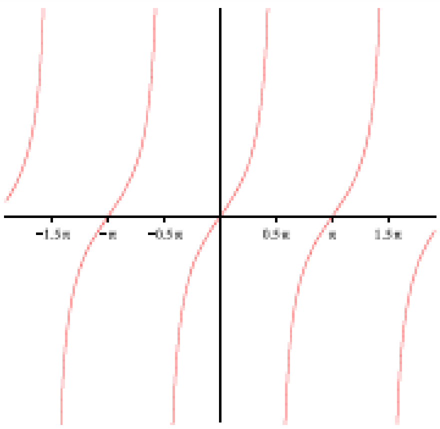
5. Graphs



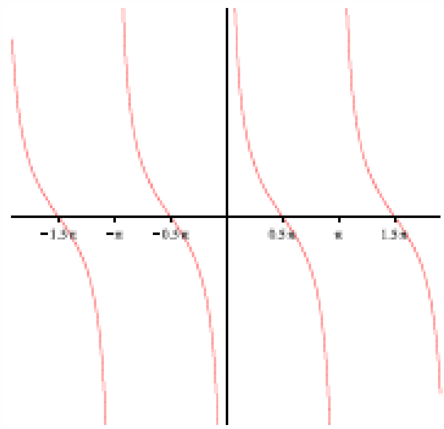
$y = \sin x$



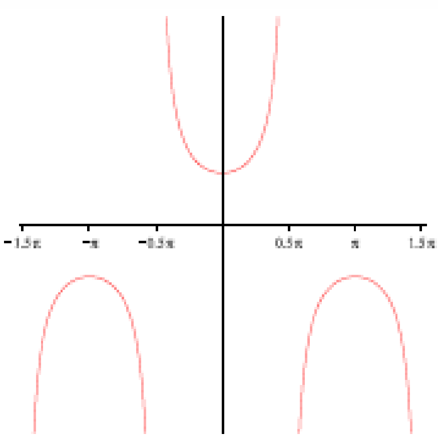
$y = \cos x$



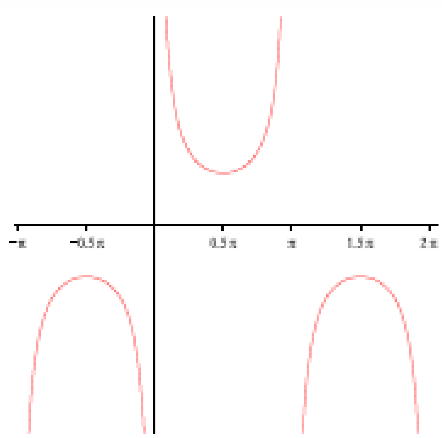
$y = \tan x$



$y = \cot x$



$y = \sec x$



$y = \csc x$

NOTE: $\leq \sin x \leq$, $\leq \cos x \leq$

Identities

1) $\sin^2\theta + \cos^2\theta = 1$

2) $\tan^2\theta + 1 = \sec^2\theta$

3) $1 + \cot^2\theta = \csc^2\theta$

4) $\sin(-\theta) = -\sin\theta$

5) $\cos(-\theta) = \cos\theta$

Add/subtract formulas

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$ $\quad$ $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

Double Angle formulas

$\sin(2x) = 2\sin x \cos x$

$$\begin{aligned}\cos(2x) &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x\end{aligned}$$

Half-Angle formulas

$$\cos^2 x = \frac{1+\cos(2x)}{2}$$

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$

ex. Solve for θ in $[0, 2\pi)$ if $\sqrt{3}\sin 2\theta + 2\sin^2\theta = 0.$

Inverse functions.

1. One-to-one functions

Def. A function f is called a **one-to-one function** if for any x_1 and x_2 in the domain:

if $x_1 \neq x_2$ then

- Horizontal Line Test

2. Inverse functions

$f^{-1}(y) = x$ if and only if

- If (x, y) is a point on the graph of $f(x)$, then

Therefore, the graph of f^{-1} is the graph of f reflected through the line

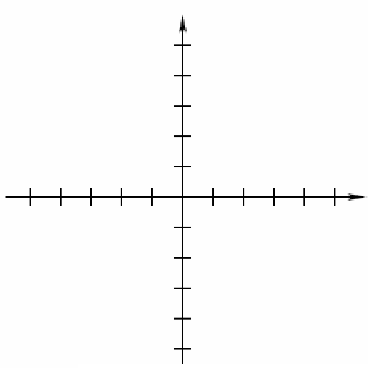
- domain and range of f^{-1}

- inverse relationships

$f^{-1}(\,f(x)\,) =$ for every x in A

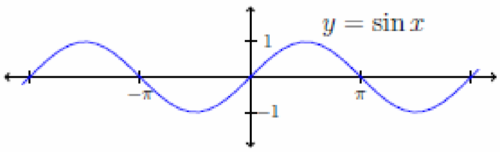
$f(\,f^{-1}(x)\,) =$ for every x in B

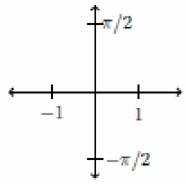
ex. Find the inverse of $f(x) = \sqrt{x+2}$. Check domain and range.



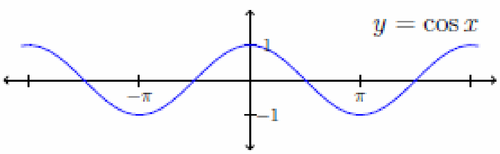
Inverse Trigonometric Functions

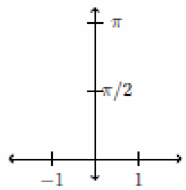
- $y = \sin^{-1} x$ if and only if



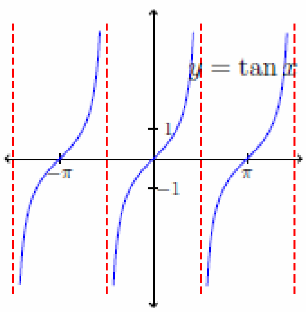


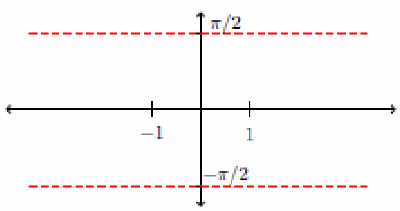
- $y = \cos^{-1} x$ if and only if





- $y = \tan^{-1} x$ if and only if





There are similar definitions for the inverse of the other trigonometric functions.

NOTE: Inverse Properties

1. $\sin(\sin^{-1} x) = x$ for $-1 \leq x \leq 1$

   $\sin^{-1}(\sin x) = x$ for $-\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}$

2. $\cos(\cos^{-1} x) = x$ for $-1 \leq x \leq 1$

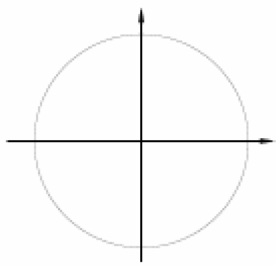
   $\cos^{-1}(\cos x) = x$ for $0 \leq x \leq \pi$

3. $\tan(\tan^{-1} x) = x$ for all x

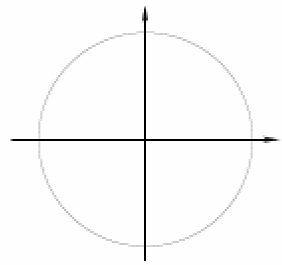
   $\tan^{-1}(\tan x) = x$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

ex. Find the following if possible:

1) $\sin^{-1}\left(-\frac{1}{2}\right)$

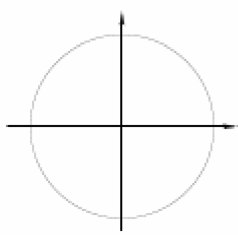


2) $\cos^{-1}\left(-\frac{1}{2}\right)$

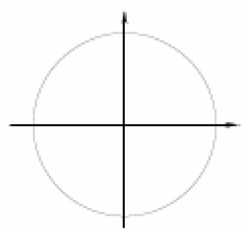


3) $\cos^{-1}(2)$

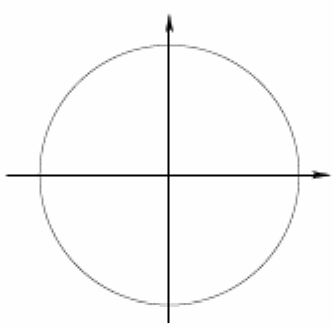
4) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$



5) $\tan^{-1}\left(\tan\frac{7\pi}{5}\right)$



ex. Use a triangle to find the exact value: $\sin(\tan^{-1}(-2))$



ex. Use a triangle to simplify the expression: $\cos(2\tan^{-1}x)$

Exponential Functions

Def. An **exponential function** with base b is a function of the form $f(x) = b^x$, where $b > 0$ and $b \neq 1$.

- Laws of Exponents ($b \neq 0$)

  1.) $b^0 =$

  2.) $b^{-x} =$

  3.) $b^{1/n} =$ $\quad$ n a positive integer

  4.) $b^x \cdot b^y =$

  5.) $\dfrac{b^x}{b^y} =$

  6.) $(b^x)^y =$

  7.) $(ab)^x =$

- If $b > 0$ and $b \neq 1$, then $y = b^x$ is a one-to-one increasing or decreasing function.

	$f(x) = b^x$	$f^{-1}(x) = \log_b(x)$
1. domain:		
2. range:		
3. intercept:		
4. asymptote:		
5. increasing if decreasing if		

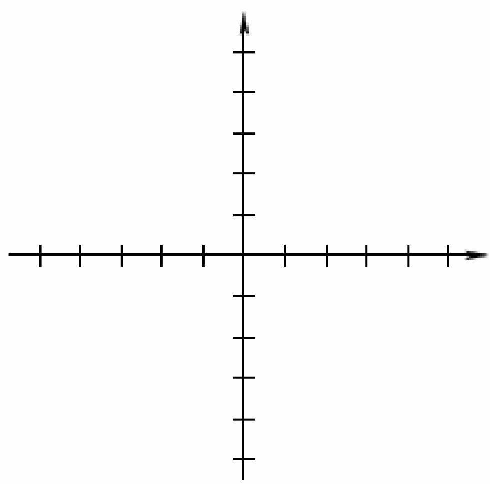
Logarithmic functions

The inverse of $y = b^x$ is the **logarithmic function** with base b, written

- $y = \log_b(x)$ if and only if
- Laws of Logarithms ($x > 0$ and $y > 0$)
  1.) $\log_b(1) =$
  2.) $\log_b(b) =$
  3.) $\log_b(xy) =$
  4.) $\log_b\left(\frac{x}{y}\right) =$
  5.) $\log_b(x^n) =$
- Inverse Properties
  1.) $\log_b(b^x) =$
  2.) $b^{\log_b x} =$
- Change of base formula

  For any $b > 0$ and $b \neq 1$, $\log_b(x) = \frac{\log_a x}{\log_a b}$

Properties of graphs of $f(x) = b^x$ and $f^{-1}(x) = \log_b(x)$:



$b > 1$

There is a base that occurs frequently in applications and is especially useful in calculus. It is the irrational number $e \approx 2.718281828459045235360287471352662 5\ldots$

Def. **Common logarithm**: $\log x =$

Def. **Natural logarithm**: $\ln x =$

ex. Evaluate:

1) $e^{-3\ln 2}$

2) $\log\left(\frac{1}{1000^2}\right)$

ex. Solve for x:

$$2\ln(x) - \ln(3 - x) = \ln\left(\frac{1}{2}\right) + \ln(8)$$

ex. Find the inverse of $f(x) = \log(x + 2) - 1$.