Rolle’s Theorem
Let $f$ be a function satisfying the following:

1) $f$ is continuous on $[a, b]$.
2) $f$ is differentiable on $(a, b)$.
3)

Then there is a number $c$ in $(a, b)$ such that

Consider the graphs:
ex. Find the value of $c$ implied by Rolle’s Theorem for $f(x) = (x^2 - 4x)^{2/3}$ on $[0, 4]$. 
We can use Rolle’s Theorem to prove the important

**Mean Value Theorem:**
Let $f$ be a function that satisfies the following conditions:

1) $f$ is continuous on $[a, b]$.

2) $f$ is differentiable on $(a, b)$.

Then there is a number $c$ in the interval $(a, b)$ such that
ex. Find the value of $c$ implied by the Mean Value Theorem for $f(x) = x^3 - x^2 - 2x$ on $[-1, 1]$. 
Theorem: If $f'(x) = 0$ for all $x$ in an interval $(a, b)$, then

Corollary: If $f'(x) = g'(x)$ for all $x$ in an interval $(a, b)$, then $f - g$ is

That is, $f(x) =$
ex. Use the theorem to verify the identity
\[ \sin^2 x + \cos^2 x = 1. \]
First Derivative Test

![Graph of a function](image)

**Increasing/Decreasing Test**

If $f'(x) > 0$ on an interval, then

If $f'(x) < 0$ on an interval, then
ex. Find the intervals on which \( f(x) = x^4 - \frac{4}{3}x^3 \) is increasing and decreasing.
ex. For which intervals is \( g(x) = \frac{x^2 - 1}{x^4} \) increasing and decreasing?
Local Extrema and the first derivative

**First Derivative Test:** Suppose that $c$ is a critical number of a continuous function $f$.

1. If $f'$ changes from positive to negative at $c$, then

2. If $f'$ changes from negative to positive at $c$, then

3. If $f'$ does not change signs at $c$, then
ex. Find all local extrema of $f(x) = x^2(x + 5)$. 
ex. Find the local maximum and minimum values of
$f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$