L11 Mean Value Theorem

Rolle's Theorem

Let f be a function satisfying the following:

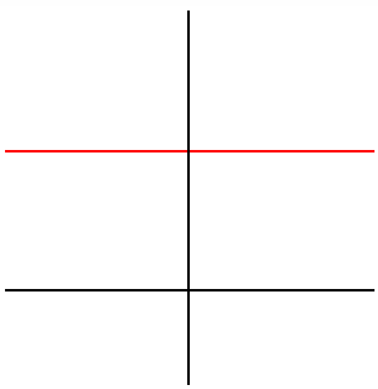
1) f is continuous on $[a, b]$.

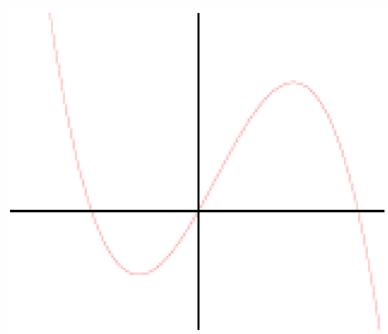
2) f is differentiable on (a, b).

3)

Then there is a number c in (a, b) such that

Consider the graphs:





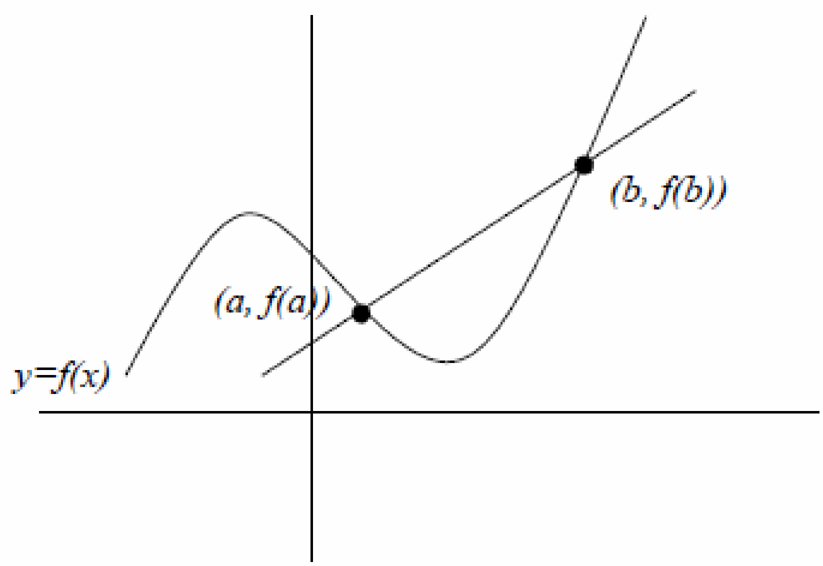
ex. Find the value of c implied by Rolle's Theorem for $f(x) = (x^2 - 4x)^{2/3}$ on $[0, 4]$.

We can use Rolle's Theorem to prove the important

Mean Value Theorem:
Let f be a function that satisfies the following conditions:

1) f is continuous on $[a, b]$.
2) f is differentiable on (a, b).

Then there is a number c in the interval (a, b) such that



ex. Find the value of c implied by the Mean Value Theorem for $f(x) = x^3 - x^2 - 2x$ on $[-1, 1]$.

Theorem: If $f'(x) = 0$ for all x in an interval (a, b), then

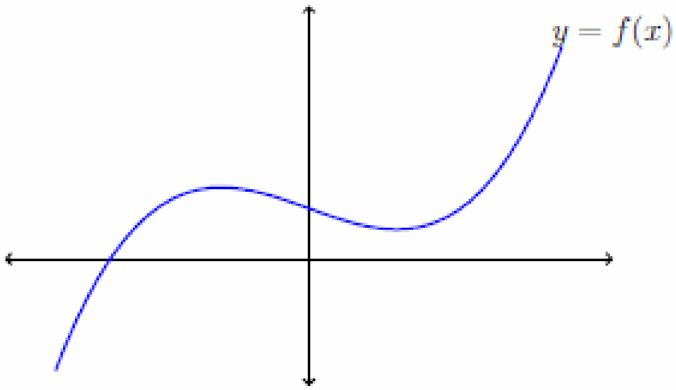
Corollary: If $f'(x) = g'(x)$ for all x in an interval (a, b), then $f - g$ is

That is, $f(x) =$

__**ex.**__ Use the theorem to verify the identity

$$\sin^2 x + \cos^2 x = 1.$$

First Derivative Test



Increasing/Decreasing Test

If $f'(x) > 0$ on an interval, then

If $f'(x) < 0$ on an interval, then

ex. Find the intervals on which $f(x) = x^4 - \frac{4}{3}x^3$ is increasing and decreasing.

<u>ex.</u> For which intervals is $g(x) = \dfrac{x^2 - 1}{x^4}$ increasing and decreasing?

Local Extrema and the first derivative

First Derivative Test: Suppose that c is a critical number of a continuous function f.

1. If f' changes from positive to negative at c, then

2. If f' changes from negative to positive at c, then

3. If f' does not change signs at c, then

ex. Find all local extrema of $f(x) = x^{\frac{2}{3}}(x+5)$.

ex. Find the local maximum and minimum values of $f(x) = \sin x + \cos x,\ 0 \leq x \leq 2\pi$