L13 Related Rates and Optimization

Related rates allow us to find the rate of change of one quantity in terms of the rate of change of a related quantity.

To solve a related rates problem:

1. Identify the desired rate of change; assign variables to all related quantities. Determine what rate(s) are given at a particular instant in time and express using your variables. Draw a sketch if possible.
2. Write an equation (mathematical model) relating the variables involved.
3. Differentiate the equation implicitly with respect to time.
4. Substitute known values and solve for the desired rate of change.

ex. A clown is blowing air into a soap bubble at the rate of 8 cm^3/sec. Assuming the bubble is spherical, how fast is the radius increasing when the volume is 288π cubic cm?

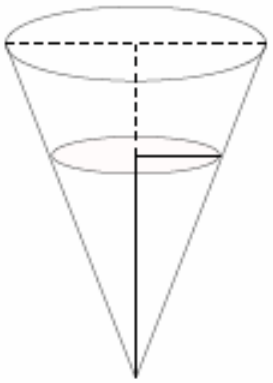
How fast is the surface area changing at that same time?

ex. A camera is mounted at a point 3000 ft from the base of a rocket launching pad.

1) If the camera to rocket distance is changing at 750 ft/sec when the rocket is 4000 ft high, how fast is the rocket rising at this time?

2) If the camera to rocket distance is changing at 750 ft/sec when the rocket is 4000 ft high, at what rate is the angle between the tip of the rocket and the camera's line of sight changing at that time?

ex. A liquid is to be cleared of sediment by pouring it through an inverted cone-shaped filter. The height of the cone is 20 inches and the diameter across the top is 16 inches. If the liquid is flowing out at 2 cubic in/min, how fast is the depth of the liquid changing when it is 12 inches deep?



ex. The base of a triangle is increasing at a rate of 2 ft/sec while the height is decreasing at a rate of 4 ft/sec. How fast is the area of the triangle changing when the base is 6 feet and the height is 10 feet?

Optimization Problems

To Solve an Optimization Problem

1. Read the problem carefully to determine the quantity that you are trying to optimize, and the conditions involved.

2. Draw a sketch if possible and assign symbols to known and unknown quantities.

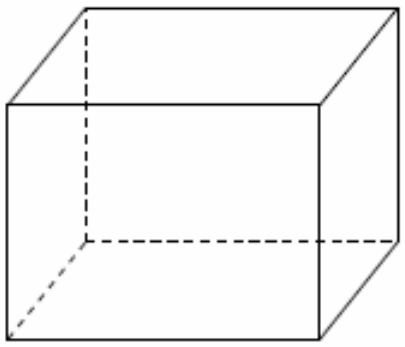
3. Find the function representing the quantity to be optimized : the Objective Function

4. If necessary, find an equation relating the variables involved (the Constraint) and write the objective function as the function of a single variable.

5. Use calculus to find the desired maximum or minimum; check your result. You can use the first or second derivative test, or the extreme value theorem if the domain is a closed interval.

ex. Find two nonnegative numbers so that the product of the first and the cube of the second is a maximum, if the sum of twice the first and three times the second is 60.

ex. A closed rectangular box is to be constructed with a surface area of 48 square feet so that its length is twice the width. What dimensions will maximize the volume of the box? What is the maximum volume?



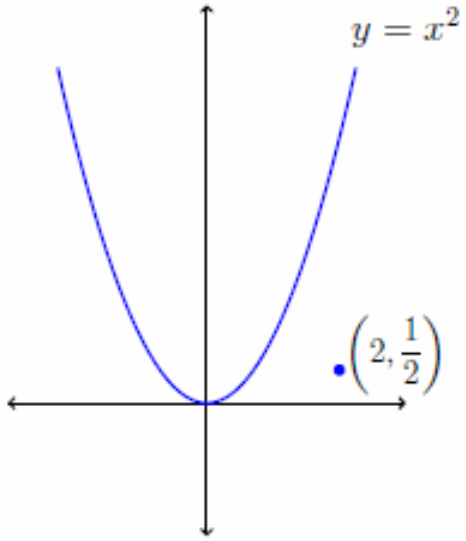
First Derivative Test
for
Absolute Extreme Values

Let c be a critical number of a continuous function f defined on an interval.

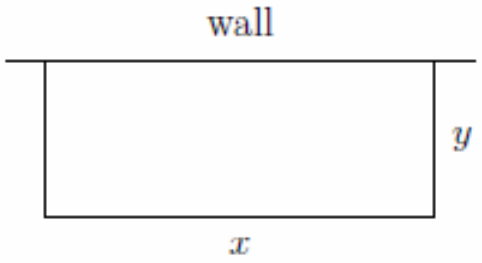
1) If $f'(x) > 0$ for all $x < c$ in the interval and $f'(x) < 0$ for all $x > c$, then

2) If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then

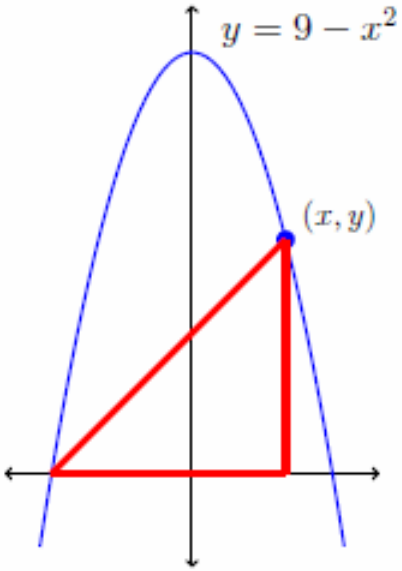
ex. Find the coordinates of the point on the curve $y = x^2$ closest to the point $\left(2, \frac{1}{2}\right)$.



ex. The management of a local Target store has decided to enclose an 800 square foot area outside the building for the garden display. One side will be formed by an external wall of the store, two sides will be constructed of pineboards costing \$6 per foot and the side opposite the store will be constructed of fencing that costs \$3 per foot. What dimensions of the enclosure will minimize the cost? Let x be the length of the side with fencing.



ex. Find the area of the largest right triangle that can be formed as shown between the x-axis and the curve $y = 9 - x^2$.



ex. A swimmer is 100 meters from a straight shore. A lifeguard is 300 meters from the point on the shore closest to the swimmer. If she can swim at 3 m/sec and run at 5 m/sec, what path will get her to the swimmer as fast as possible?