L17 The Substitution Method

Consider \( \int 6x^2(2x^3 - 1)^{20} \, dx \).

How to evaluate?
Substitution Rule

If $u = g(x)$ is a differentiable function whose range is an interval $I$ and $f$ is continuous on $I$, then

$$\int f(g(x))g'(x) \, dx = \int f(u) \, du$$

ex. $\int (3x^2 + 1)(x^3 + x - 2)^9 \, dx =$
Evaluate the integrals:

ex. \[ \int \frac{\cos x + 2}{\sin x + 2x} \, dx \]

ex. \[ \int (8x^2 - 4) \sqrt{2x^3 - 3x} \, dx = \]
\textbf{ex.} \quad \int \frac{e^{1 + \frac{1}{x}}}{x^2} \, dx =

Note the following result:

\textbf{ex.} If \( a \) is any nonzero number, \( \int e^{ax} \, dx = \)
ex. $\int \frac{\sin x}{\cos^3 x} \, dx =$
ex. Find a formula for $\int \tan x \, dx$
Some integrals require an additional step:

\[ \text{ex. } \int (1 + x)\sqrt{2 - x} \, dx = \]
ex. Evaluate: \[ \int \frac{e^{3x}}{1 + e^x} \, dx \]
ex. \[ \int \frac{1}{x(ln \ x)^2} \ dx = \]

ex. Find the area of the region bounded by the \( x \)-axis and \[ f(x) = \frac{1}{x(ln \ x)^2} \] from \( x = e \) to \( x = e^4 \).
Substitution Rule for Definite Integrals

If $g'(x)$ is continuous on $[a, b]$ and $f$ is continuous on the range of $g$, then

$$\int_{a}^{b} f(g(x))g'(x) \, dx = \int_{g(a)}^{g(b)} f(u) \, du$$

Use this rule to find the area above.
Integrals of symmetric functions

Suppose $f$ is continuous on $[-a, a]$.

a) If $f$ is even, then $\int_{-a}^{a} f(x) \, dx =$

b) If $f$ is odd, then $\int_{-a}^{a} f(x) \, dx =$
\textbf{ex.} Evaluate: $\int_{-2}^{2} x \sqrt{4 - x^2} \, dx$

\textbf{ex.} Find the area between the $x$-axis and $f(x) = x^4 - 4x^2$ from $x = -2$ to $x = 2$. 