L17 The Substitution Method

Consider $\displaystyle\int 6x^2(2x^3-1)^{20}\,dx$.

How to evaluate?

Substitution Rule

If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

<u>**ex.**</u> $\int (3x^2+1)(x^3+x-2)^9\,dx =$

Evaluate the integrals:

ex. $$\int \frac{\cos x + 2}{\sin x + 2x}\, dx$$

ex. $$\int (8x^2 - 4)\sqrt{2x^3 - 3x}\, dx =$$

ex. $\displaystyle\int \frac{e^{1+\frac{1}{x}}}{x^2}\,dx =$

Note the following result:

ex. If a is any nonzero number, $\displaystyle\int e^{ax}\,dx =$

ex. $\displaystyle\int \frac{\sin x}{\cos^3 x}\,dx =$

ex. Find a formula for $\int \tan x \, dx$

Some integrals require an additional step:

<u>**ex.**</u> $\displaystyle\int (1+x)\sqrt{2-x}\,dx =$

ex. Evaluate: $\displaystyle\int \frac{e^{3x}}{1+e^x}\,dx$

ex. $\displaystyle\int \frac{1}{x(\ln x)^2}\,dx =$

ex. Find the area of the region bounded by the x-axis and $f(x) = \dfrac{1}{x(\ln x)^2}$ from $x = e$ to $x = e^4$.

Substitution Rule for Definite Integrals

If $g'(x)$ is continuous on $[a, b]$ and f is continuous on the range of g, then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Use this rule to find the area above.

Integrals of symmetric functions

Suppose f is continuous on $[-a, a]$.

a) If f is even, then $\int_{-a}^{a} f(x)\,dx =$

b) If f is odd, then $\int_{-a}^{a} f(x)\,dx =$

ex. Evaluate: $\int_{-2}^{2} x\sqrt{4-x^2}\,dx$

ex. Find the area between the x-axis and $f(x) = x^4 - 4x^2$ from $x = -2$ to $x = 2$.