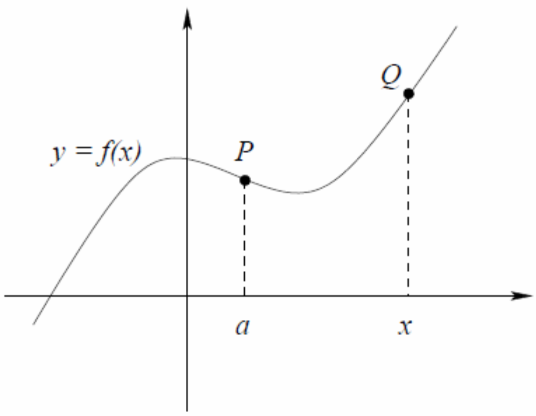
L5 Definition of the derivative

Tangent Lines and Slope



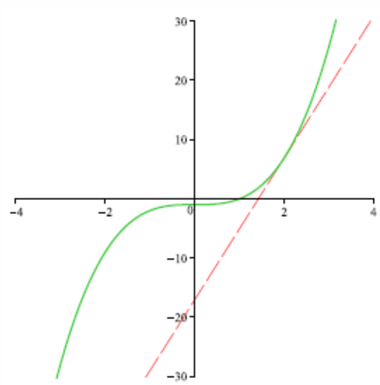
Def. The slope of the secant line through the point $P(a, f(a))$ and a nearby point $Q(x, f(x))$:

Def. The **tangent line** to $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$m =$

provided that the limit exists.

ex. Find the equation of the tangent line to $f(x) = x^3 - 1$ at $x = 2$.



Alternate Definition of the Slope of a Tangent Line

Let $h = x - a$. Then

and $m =$

ex. Find the slope of the tangent line to

$f(x) = \dfrac{x}{x+1}$ at $x = 2$.

NOTE: We can use our formulas to find the slope of the tangent line for $f(x)$ at any point $(a, f(a))$.

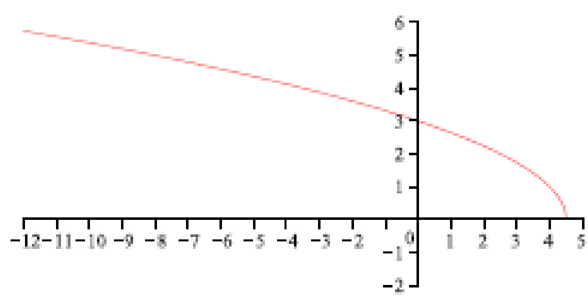
ex. Find the slope of the tangent line to $f(x) = \sqrt{9 - 2x}$ at $x = a$.

We can use this formula to find the slope of the tangent line to $f(x) = \sqrt{9 - 2x}$ at a given value $x = a$ (if the limit exists):

1) $x = -8$

2) $x = 0$

3) $x = \dfrac{9}{2}$



The derivative as a function

Def. Given $y = f(x)$,

$$f'(x) =$$

The derivative is itself a function of f. Its domain:

Other notations for the derivative:

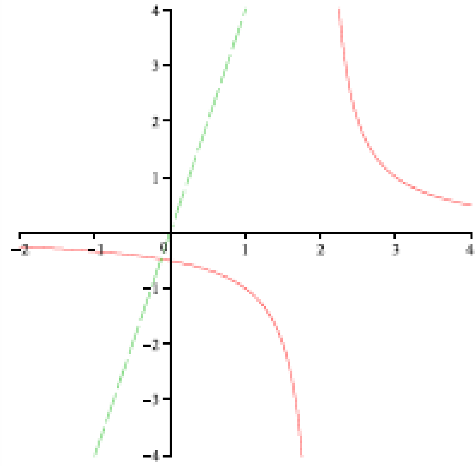
Process of finding the derivative is called

__**ex.**__ Find the function $f'(x)$ for $f(x) = \frac{1}{x-2}$.
What is its domain?

__**Def.**__ A function f is **differentiable** at $x = a$ if $f'(a)$ exists. It is differentiable on an open interval if it is differentiable at each number in the interval.

__**ex.**__ Find each interval for which $f(x) = \frac{1}{x-2}$ is differentiable.

ex. Find each x-value at which the tangent line to the graph of $f(x) = \dfrac{1}{x-2}$ is perpendicular to the line $y = 4x$.



Recall: the **derivative** of a function f at $x = a$ is

$$f'(a) =$$

The derivative as a function

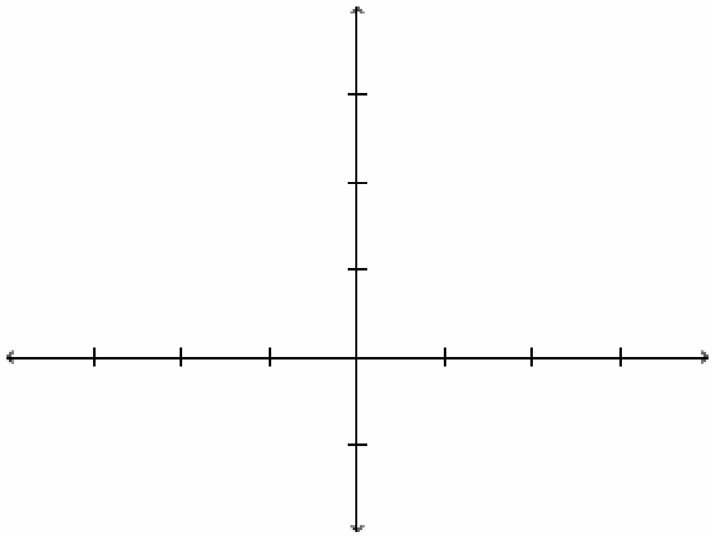
Def. Given $y = f(x)$, $f'(x) =$

ex. Let $f(x) = \begin{cases} 1 - x & x < 1 \\ x^2 - 1 & x \geq 1 \end{cases}$.

1) Find $f'(1)$ if possible.

2) Find a formula for $f'(x)$, where $f(x) = \begin{cases} 1 - x & x < 1 \\ x^2 - 1 & x \geq 1 \end{cases}$.

3) Sketch the graph of $f(x) = \begin{cases} 1 - x & x < 1 \\ x^2 - 1 & x \geq 1 \end{cases}$.



What do you note about continuity and differentiability of $f(x)$ at $x = 1$?

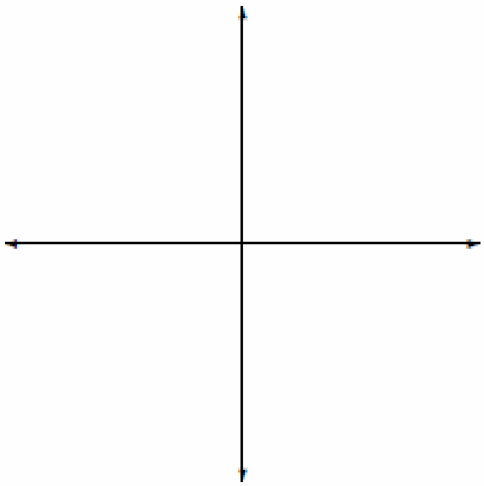
Theorem. If f is differentiable at $x = a$, then f is continuous at $x = a$.

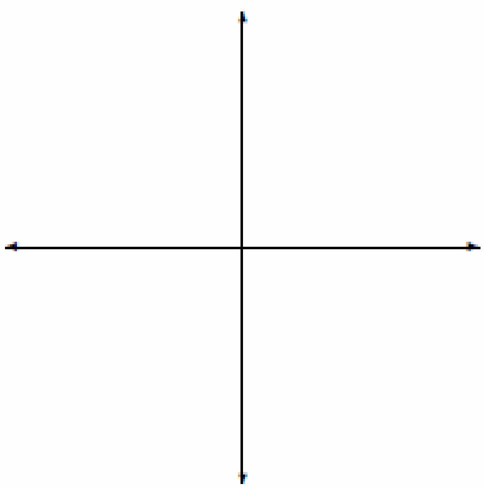
ex. Find $f'(1)$ if $f(x) = (x-1)^{1/3}$.

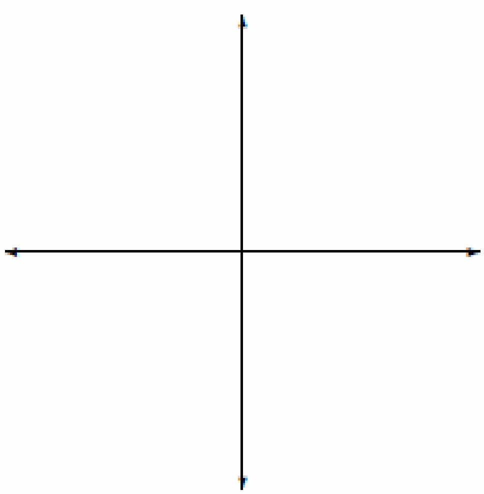
Def. The graph of a function $f(x)$ has a **vertical tangent line** at $x = a$ if

What about a **horizontal tangent line**?

When is a function not differentiable at a point?

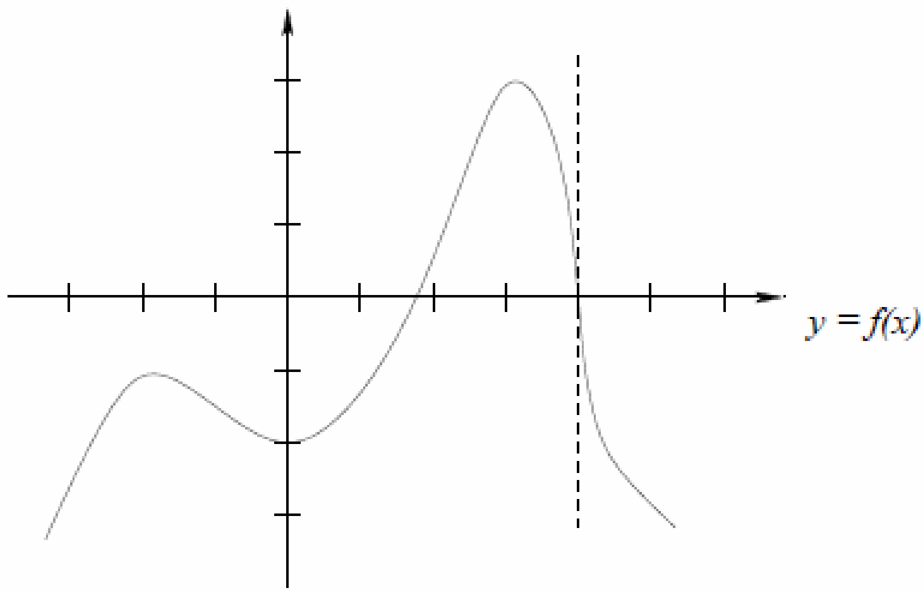


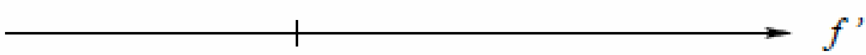


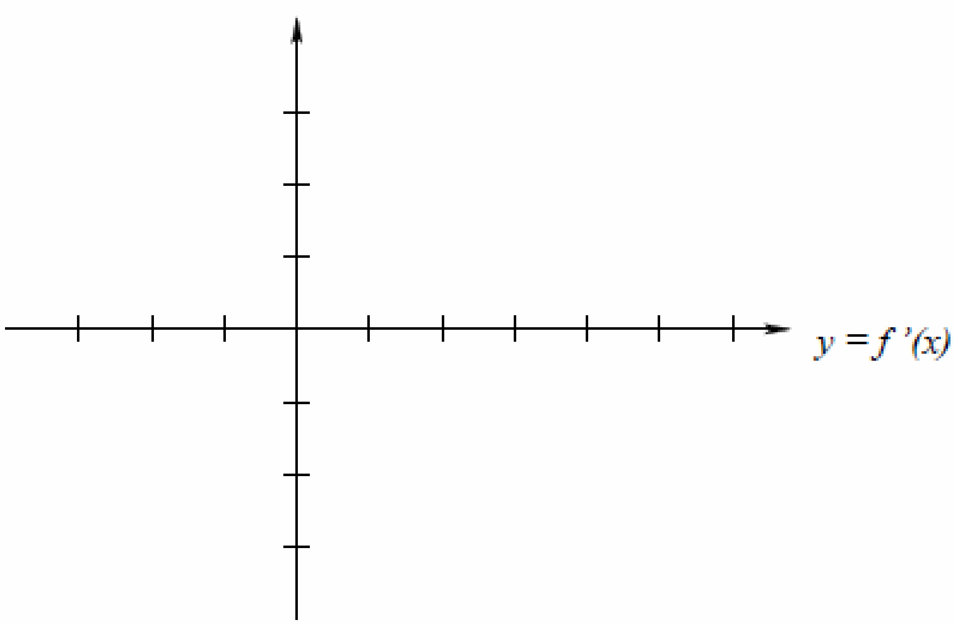


We know that at $x = a$, $f'(a)$ gives the slope of the tangent line to the graph of f at the point $(a, f(a))$.

ex. Given the graph of $f(x)$, sketch a possible graph of its derivative.







ex. Find all points P on the parabola $y = x^2$ such that the tangent line at P passes through the point $(0, -4)$.