L6 Basic rules of differentiation: polynomials and exponentials

Derivative of a Constant

If \( c \) is a constant, then \( \frac{d}{dx}(c) = \)

Power functions of the form \( f(x) = x^n \)

1) \( \frac{d}{dx}(x) = \)
2) If $n$ is a positive integer, $\frac{d}{dx}(x^n) =$

To prove this, we need the formula

\[ x^n - a^n = (x - a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \ldots + xa^{n-2} + a^{n-1}) \]

This result extends to all real numbers.

**Power Rule**

For any real number $r$, $\frac{d}{dx}(x^r) =$
**ex.** Find the following derivatives:

1) \[ \frac{d}{dx} \left( \frac{\pi}{2} \right) = \]

2) \[ \frac{d}{dx} (x^{125}) = \]

3) \[ \frac{d}{dx} \left( \frac{1}{x^5} \right) = \]

**ex.** Find each \( x \)-value at which \( f(x) = x\sqrt{x} \) is perpendicular to \( 2y + x = 6 \).
The next rules, based on the limit laws, allow us to find derivatives of some combinations of functions.

**Constant Multiple Rule**

If \(c\) is a constant and \(f\) is differentiable then

\[
\frac{d}{dx}(cf(x)) =
\]

**Sum and Difference Rules**

If \(f\) and \(g\) are both differentiable,

\[
\frac{d}{dx}[f(x) \pm g(x)] =
\]

We can now find the derivative of any polynomial function.

**ex.** Find \(f'(x)\) if \(f(x) = 4x^3 + 9x^2 - 12x + 3\).
ex. At which $x$-values does $f(x) = 4x^3 + 9x^2 - 12x + 3$ have horizontal tangent lines? Write the equation of each line.
\[ f(x) = 4x^3 + 9x^2 - 12x + 3 \]
**Def.** The **normal line** to a curve at a point \( P \) is the line through \( P \) that is perpendicular to the tangent line at \( P \).

**ex.** Find the equation of the normal line to
\[
f(x) = \frac{\sqrt{x} - 6\sqrt[3]{x} + 4}{\sqrt[4]{x}} \quad \text{at } x = 1.
\]
Derivatives of Exponential Functions

Consider \( \frac{d}{dx} f(x) \) where \( f(x) = a^x, \ a > 0. \)

First, find the value of \( f'(0) \).

\[
f'(0) =
\]

To find \( f'(x) \):
What does this say about the rate of change of any exponential?

If \( a = 2 \), \( f'(0) = \lim_{h \to 0} \)

If \( a = 3 \), \( f'(0) = \lim_{h \to 0} \)

**Def.** \( e \) is the number such that \( \lim_{h \to 0} \frac{e^h - 1}{h} = \)

We have: \( \frac{d}{dx}(e^x) = \)
ex. Find $g'(x)$ if $g(x) = ex^2 + 2e^x + xe^2 + xe^2$. 
Product and Quotient Rules

ex. Let $f(x) = x^2$ and $g(x) = x + 1$.

What is $\frac{d}{dx}[f(x)g(x)]$?

The Product Rule

If $f$ and $g$ are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] =$$
ex. Find each point at which \( f(x) = xe^x \) has a horizontal tangent line.

**The Quotient Rule:** If \( f \) and \( g \) are both differentiable, then

\[
\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) =
\]
ex. Find $f'(x)$ if $f(x) = \frac{4x}{x^2 + 1}$.

Find the equation of all horizontal tangent lines of the graph of $f(x)$. 
**ex.** Find the equation of the normal line to \( y = \frac{1}{x^2 - 2} \) at \( x = 2 \).

**ex.** Find \( f'(4) \) if \( f(x) = \frac{(\sqrt{x} - 2)^2}{\sqrt{x}} \).
example. If \( h(x) = \frac{x^2 - 3}{xf(x)} \), \( f(-2) = 3 \), and \( f'(-2) = \frac{1}{2} \), find \( h'(-2) \).
ex. At what point(s) do the tangent lines to $y = \frac{x^3 + x^2}{x}$ pass through the point $(2, -3)$?