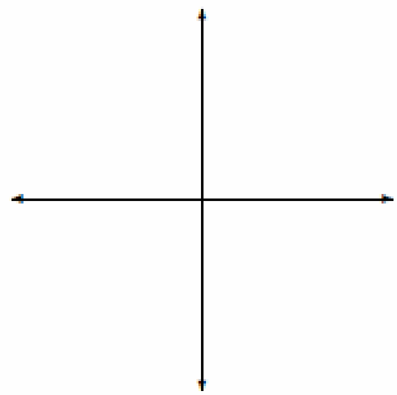
L6 Basic rules of differentiation: polynomials and exponentials

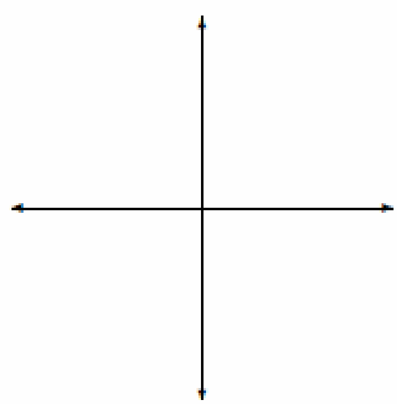
Derivative of a Constant

If c is a constant, then $\frac{d}{dx}(c) =$



Power functions of the form $f(x) = x^n$

1) $\frac{d}{dx}(x) =$



2) If n is a positive integer, $\frac{d}{dx}(x^n) =$

To prove this, we need the formula

$$x^n - a^n =$$
$$(x-a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + ... + xa^{n-2} + a^{n-1})$$

This result extends to all real numbers.

Power Rule

For any real number r, $\frac{d}{dx}(x^r) =$

ex. Find the following derivatives:

1) $\dfrac{d}{dx}\left(\dfrac{\pi}{2}\right) =$

2) $\dfrac{d}{dx}(x^{125}) =$

3) $\dfrac{d}{dx}\left(\dfrac{1}{x^5}\right) =$

ex. Find each x-value at which $f(x) = x\sqrt{x}$ is perpendicular to $2y + x = 6$.

The next rules, based on the limit laws, allow us to find derivatives of some combinations of functions.

Constant Multiple Rule

If c is a constant and f is differentiable then

$$\frac{d}{dx}(cf(x)) =$$

Sum and Difference Rules

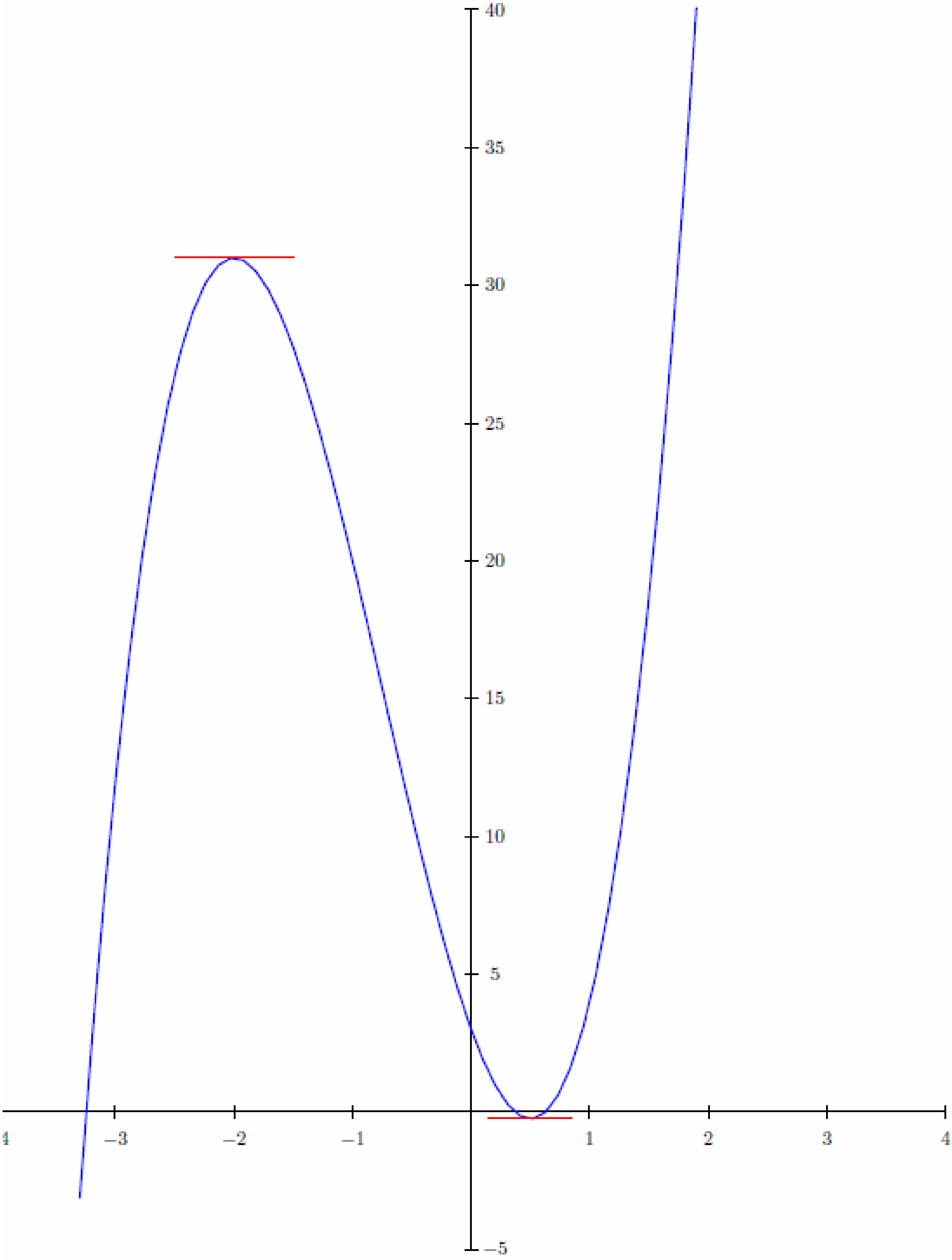
If f and g are both differentiable,

$$\frac{d}{dx}[f(x) \pm g(x)] =$$

We can now find the derivative of any polynomial function.

ex. Find $f'(x)$ if $f(x) = 4x^3 + 9x^2 - 12x + 3$.

ex. At which x-values does $f(x) = 4x^3 + 9x^2 - 12x + 3$ have horizontal tangent lines? Write the equation of each line.



$$f(x) = 4x^3 + 9x^2 - 12x + 3$$

<u>Def.</u> The **normal line** to a curve at a point P is the line through P that is perpendicular to the tangent line at P.

<u>ex.</u> Find the equation of the normal line to

$$f(x) = \frac{\sqrt{x} - 6\sqrt[3]{x} + 4}{\sqrt[4]{x}} \text{ at } x = 1.$$

Derivatives of Exponential Functions

Consider $\frac{d}{dx}f(x)$ where $f(x) = a^x$, $a > 0$.

First, find the value of $f'(0)$.

$f'(0) =$

To find $f'(x)$:

What does this say about the rate of change of any exponential?

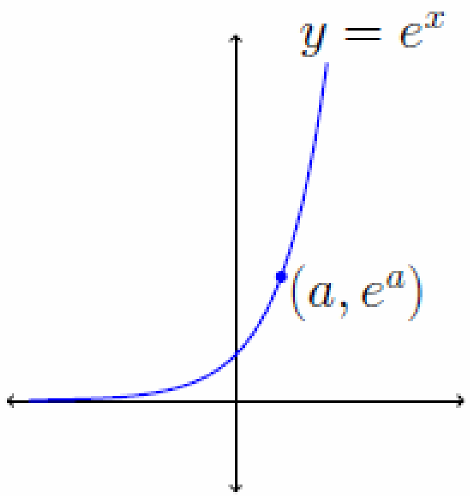
If $a = 2$, $f'(0) = \lim_{h \to 0}$

If $a = 3$, $f'(0) = \lim_{h \to 0}$

<u>Def.</u> e is the number such that $\lim_{h \to 0} \frac{e^h - 1}{h} =$

We have: $\frac{d}{dx}(e^x) =$

NOTE:



<u>**ex.**</u> Find $g'(x)$ if $g(x) = ex^2 + 2e^x + xe^2 + x^{e^2}$.

Product and Quotient Rules

ex. Let $f(x) = x^2$ and $g(x) = x + 1$.

What is $\frac{d}{dx}[f(x)g(x)]$?

The Product Rule

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] =$$

ex. Find each point at which $f(x) = xe^x$ has a horizontal tangent line.

The Quotient Rule: If f and g are both differentiable, then

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =$$

ex. Find $f'(x)$ if $f(x) = \dfrac{4x}{x^2+1}$.

Find the equation of all horizontal tangent lines of the graph of $f(x)$.

ex. Find the equation of the normal line to $y = \dfrac{1}{x^2 - 2}$ at $x = 2$.

ex. Find $f'(4)$ if $f(x) = \dfrac{(\sqrt{x} - 2)^2}{\sqrt{x}}$.

ex. If $h(x) = \dfrac{x^2 - 3}{xf(x)}$, $f(-2) = 3$, and $f'(-2) = \dfrac{1}{2}$,

find $h'(-2)$.

ex. At what point(s) do the tangent lines to $y = \dfrac{x^3 + x^2}{x}$ pass through the point $(2, -3)$?