L7 Derivatives of Trigonometric Functions

Recall: $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} =$

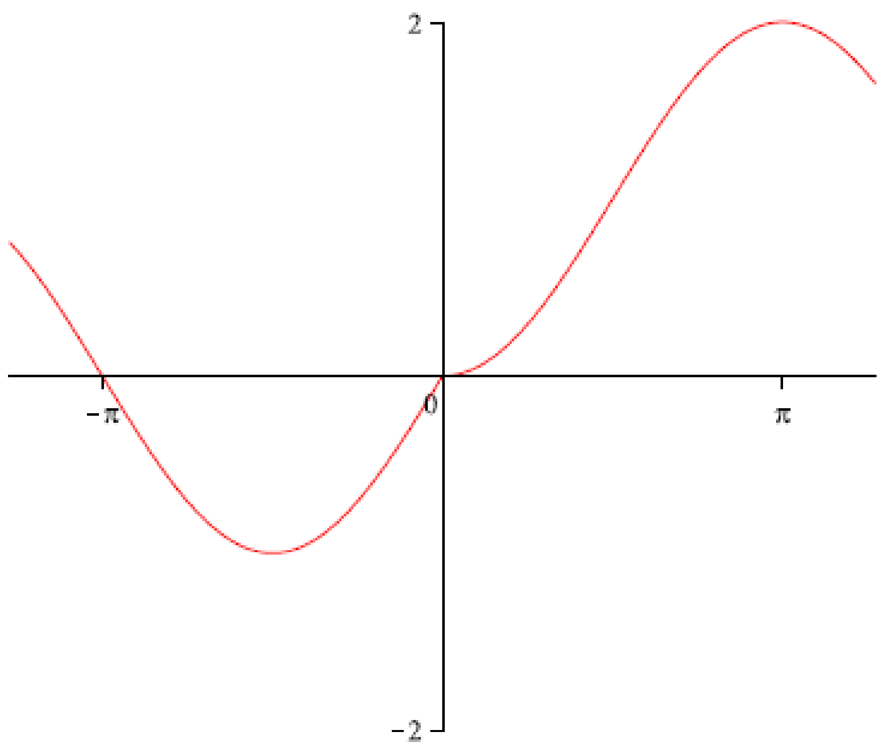
$$\lim_{\theta \to 0} \frac{\tan\theta}{\theta} =$$

$$\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} =$$

ex. Let $f(x) = \begin{cases} \sin x & x < 0 \\ 1 - \cos x & x \geq 0 \end{cases}$.

Is $f(x)$ continuous at $x = 0$?

Find $f'(0)$ if possible, where $f(x) = \begin{cases} \sin x & x < 0 \\ 1 - \cos x & x \geq 0 \end{cases}$.



$f(x) = \begin{cases} \sin(x) & x < 0 \\ 1-\cos(x) & x \geq 0 \end{cases}$

1) $\dfrac{d}{dx}(\sin x) =$

2) $\dfrac{d}{dx}(\cos x) =$

3) $\dfrac{d}{dx}(\tan x) =$

4) $\dfrac{d}{dx}(\cot x) =$

5) $\frac{d}{dx}(\sec x) =$

6) $\frac{d}{dx}(\csc x) =$

ex. Find the slope of the tangent line to $f(x) = \sin x \cos x$ at $x = \dfrac{\pi}{3}$.

ex. Let $f(x) = \dfrac{1 - \tan x}{\sec x}$. Find $f'(x)$ and each x-value for which the graph of f has a horizontal tangent line.

ex. If $f(x) = \dfrac{\sec x}{e^x}$, find the slope of the tangent line at $x = -\dfrac{\pi}{4}$.

Theorem (The Chain Rule):

If g is differentiable at x and f is differentiable at $g(x)$, then the composite function

$F = f \circ g = f(g(x))$ is differentiable and

$F'(x) =$

Chain Rule (second version: rate of change)
If $y = f(u)$ and $u = g(x)$ are differentiable functions, then

ex. Find $h'(x)$ for $h(x) = \sqrt{x^2 + 2x - 3}$.

ex. If $f(x) = (2e^x - 3x)^5$, find $f'(0)$.

Power Rule combined with the Chain Rule

If n is any real number and $u = g(x)$ is differentiable, then

or

ex. If $g(x) = \dfrac{4}{\sqrt[4]{(3-2x^2)^3}}$ find $g'(x)$.

ex. If $f(x) = (2x+3)^4\ (2-x)^3$, find $f'(x)$. At which x-values does the graph of $f(x)$ have a horizontal tangent line?

__**ex.**__ Find and simplify $f'(x)$ if $f(x) = \dfrac{x^2}{\sqrt{3-2x}}$. Find the slope of the tangent line to $f(x)$ at $x = 1$.

Derivatives involving Exponential Functions

Recall that $\dfrac{d}{dx}(e^x) =$

If $u = f(x)$ is differentiable, we can apply the Chain Rule to find $\dfrac{d}{dx}(e^u)$:

ex. If $f(x) = e^{x+2\cos x}$, find $f'(x)$.
Then find each x-value at which $f(x)$ has a horizontal tangent line on $[0, 2\pi]$.

ex. Find $f'(x)$ if $f(x) = e^{ax}$ for any constant a.

ex. Evaluate: $\dfrac{d}{dx}\left[\dfrac{(e^x - 2)^2}{e^x}\right]$

Derivatives involving trigonometric functions

<u>ex.</u> Find $f'(x)$ for

a) $f(x) = \tan(x^3)$

b) $f(x) = \tan^3(x)$

<u>ex.</u> Find the derivative of $f(x) = \sec^2(\sin 4x)$.

General Power Rule: $\frac{d}{dx}[u^n] = nu^{n-1}\frac{du}{dx}$

$\frac{d}{dx}(e^x) =$

$\frac{d}{dx}(e^u) =$

$\frac{d}{dx}(\sin x) =$

$\frac{d}{dx}(\sin u) =$

$\frac{d}{dx}(\cos x) =$

$\frac{d}{dx}(\cos u) =$

$\frac{d}{dx}(\tan x) =$

$\frac{d}{dx}(\tan u) =$

$\frac{d}{dx}(\sec x) =$

$\frac{d}{dx}(\sec u) =$