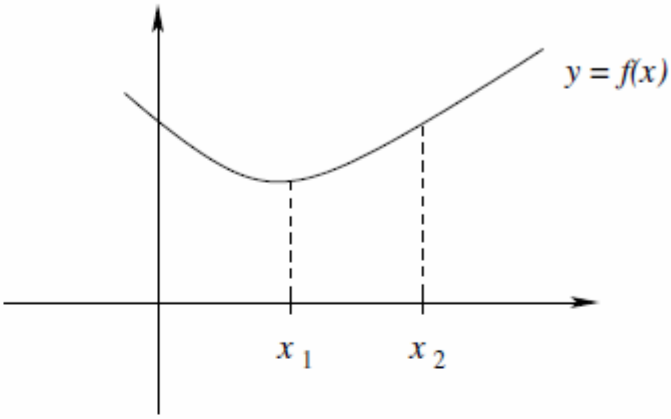
L8 Rates of Change and Higher Derivatives

Average Rate of Change



Instantaneous Rate of Change

If $s = f(t)$ is the position of a particle moving in a straight line, then

ex. Suppose the position of a particle is given by

$$s = f(t) = 2t^3 - 15t^2 + 24t,$$

where t is measured in seconds and s in feet.

a) Find the velocity of the particle at any time t.

b) Find the velocity at $t = 3$ seconds.

c) When is the particle at rest?

d) When is the particle moving in a positive direction?

e) Draw a diagram to represent the particle's motion.

f) Find the total distance the particle moves in the first six seconds.

Higher Derivatives

If $y = f'(x)$ is differentiable, we can find its derivative, a new function called $f''(x)$.

The limit definition:

In the same way, the derivative of $f''(x)$ is $f'''(x)$, and in general, we denote the nth derivative of f as $f^n(x)$.

Other notation:

The second derivative plays an important role: It is the rate at which f' changes.

Acceleration

g) Find the acceleration of $s(t) = 2t^3 - 15t^2 + 24t$ at any time t.

h) When is the particle **speeding up** and when is it **slowing down**?