L8 Rates of Change and Higher Derivatives

Average Rate of Change

Instantaneous Rate of Change
If \( s = f(t) \) is the position of a particle moving in a straight line, then

**ex.** Suppose the position of a particle is given by

\[
s = f(t) = 2t^3 - 15t^2 + 24t,
\]

where \( t \) is measured in seconds and \( s \) in feet.

a) Find the velocity of the particle at any time \( t \).

b) Find the velocity at \( t = 3 \) seconds.

c) When is the particle at rest?

d) When is the particle moving in a positive direction?
e) Draw a diagram to represent the particle’s motion.

f) Find the total distance the particle moves in the first six seconds.
Higher Derivatives

If \( y = f'(x) \) is differentiable, we can find its derivative, a new function called \( f''(x) \).

The limit definition:

In the same way, the derivative of \( f''(x) \) is \( f'''(x) \), and in general, we denote the \( n \)th derivative of \( f \) as \( f^n(x) \).

Other notation:
The second derivative plays an important role: It is the rate at which $f'$ changes.

Acceleration

g) Find the acceleration of $s(t) = 2t^3 - 15t^2 + 24t$ at any time $t$. 
h) When is the particle speeding up and when is it slowing down?