MAC2311 Practice Exam 1.   Name: **ONEHUNDRED PERCENT**

Questions 1-11 are 6 points each.

1. Find each x-value on $[0, 2\pi)$ at which $f(x) = \frac{\sin x}{\cos x + \sin 2x}$ has a vertical asymptote

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2. Solve the inequality: $\frac{x^2}{x+6} \geq 3$

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3. Let $f(x) = \frac{2}{x}$ and $g(x) = \frac{x}{x+1}$. Find $f \circ g$ and its domain.

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4. The Intermediate Value Theorem guarantees a solution to $y = x^3 - 15x + 1$ in which intervals?

a. $[-2, 1]$ b. $[-1, 0]$ c. $[0, 1]$ d. $[1, 2]$ e. $[2, 3]$

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5. Evaluate $\lim\limits_{x \to 0} \frac{1 - \cos(4x)}{x^2}$

6. If $f(x) = \frac{x^2 - 2x}{|x|}$ which of the following statements are true?

P. $\lim\limits_{x \to 0^-} f(x) = -2$

Q. f has a removable discontinuity at $x = 0$

R. $\lim\limits_{x \to -\infty} f(x) = -\infty$

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7. For what value of a is $f(x) = \begin{cases} \frac{x}{a} & x \le 2 \\ \frac{\frac{1}{x+1} - \frac{1}{3}}{x-2} & x > 2 \end{cases}$ continuous at $x = 2$?

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8. Find $f''(1)$ for $f(x) = x^4 e^x$

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9. Find the equation of the tangent line to $x^4 + y^4 = 16$ at $(1, \sqrt{3})$

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10. Find the derivative of $y = x^{\cos x}$ using logarithmic differentiation.
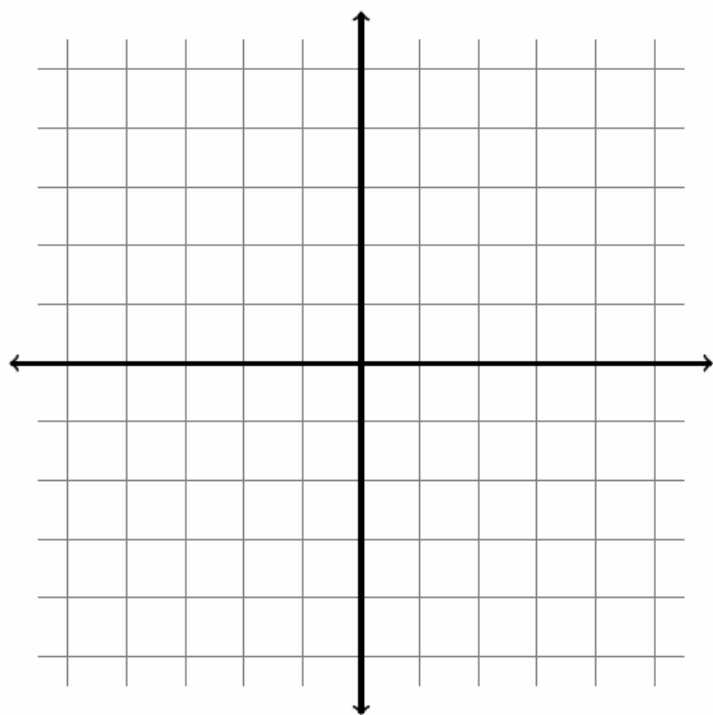
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11. Find the derivative of $y = x^3 \sin^{-1} x$

Show all work for full credit.

1. Part a is 6 points, part b is 6 points, and part c is 3 points.

$$\text{Let } f(x) = \begin{cases} x^2 + 4x & x < 0 \\ 1 - |x - 1| & 0 < x < 3 \\ \ln(x - 3) & x > 3 \end{cases}$$

(a) Sketch the graph of $f(x)$.

(b) Find the limits:

1) $\lim\limits_{x \to 3^-} f(x) = $ _____

2) $\lim\limits_{x \to 3^+} f(x) = $ _____

3) $\lim\limits_{x \to 3} f(x) = $ _____

4) $\lim\limits_{x \to 0} f(x) = $ _____

5) $\lim\limits_{x \to 1} f(x) = $ _____

6) $\lim\limits_{x \to \infty} f(x) = $ _____

(c) List all discontinuities of $f(x)$ and state whether they are jump, infinite, or removable.

2. The position of a ball thrown in the air in feet after t seconds is given by the function

$$s(t) = 40t - 16t^2, \ t \geq 0$$

(a) Find a formula for the average velocity of the ball on the interval from $t = 2$ to $t = 2 + h$ for $h \neq 0$ (4 points)

Average velocity = _____

(b) Use a limit to find the velocity of the ball at the instant $t = 2$ and include units in your answer (3 points)

Instantaneous velocity = _____

3. Evaluate the limit (8 points)

$$\lim_{x \to -\infty} \frac{\sqrt{9x^2 - 5}}{2 - x}$$

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4. The position in meters of a particle moving along the s-axis after t seconds is

$$s(t) = \tfrac{1}{3}t^3 - \tfrac{5}{2}t^2 + 6t \text{ for } t \geq 0.$$

(a) Draw sign diagrams for the velocity and acceleration of the particle (4 points)

(b) When is the particle moving in a positive direction? (2 points)

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(c) When is the particle moving in a negative direction? (2 points)

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(d) When is the particle speeding up? (2 points)

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(e) When is the particle slowing down? (2 points)

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