MAC2311 Practice Exam 2. Name: ONEHUNDED PERCENT

1. The area under the curve $y = f(x)$ and above the $x-$axis on [0,1] is represented by $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt[3]{\frac{i}{n}}\frac{1}{n}$. Find the area.

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2. Find min and max values of $y = \sin^2 x + \sin x$ on $[0,2\pi]$

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3. Find critical numbers of $y = x^2 e^x$

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4. Evaluate $\int_0^1 \frac{x^2-1}{x^4-1}\,dx$

5. Evaluate $\int \frac{3x^2}{x^3-4}\,dx$

6. Evalute $\frac{d}{dx}\int_0^{x^2} \frac{t^3}{t^3+2}\,dt$

7. A soft drink dispenser pours a soft drink at the rate of $f(t) = \frac{20t}{1+2t^2}$ml/sec where t is the elapsed time in seconds. How much of the soft drink is dispensed in the first three seconds?

8. Consider the region between the curve $f(x) = 2 + 6x^2$ and the x-axis on the interval [0,2]. Calculate R_4, find the Reimann Sum approximation R_n, simplify it, and find the exact area by taking the limit of R_n as $n \to \infty$

9. Sketch a graph of $f(x) = \frac{x^2-9}{x^2-x-2}$

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10. Evaluate $\lim\limits_{x\to\infty} (\frac{x}{1-x})^x$

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11. Let $G(x) = \int_{-2}^{x} g(t)\, dt$ where $g(t) = \frac{t(16-t^2)}{1+t^4}$. Which of the following statements is/are true?

P. $G(x)$ is increasing on the interval (-2,2)
Q. $G(x)$ is an antiderivative of $g(x)$
R. $G(x)$ has a local maximum at $x = 4$

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12. Find the value of the Mean Value Theorem implied by $f(x) = x - \ln x$ on the interval $[1,\ e]$

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13. On which intervals is $f(x) = \cos^2 x - 2\sin x$ concave up on the interval $[0,\ 2\pi)$?

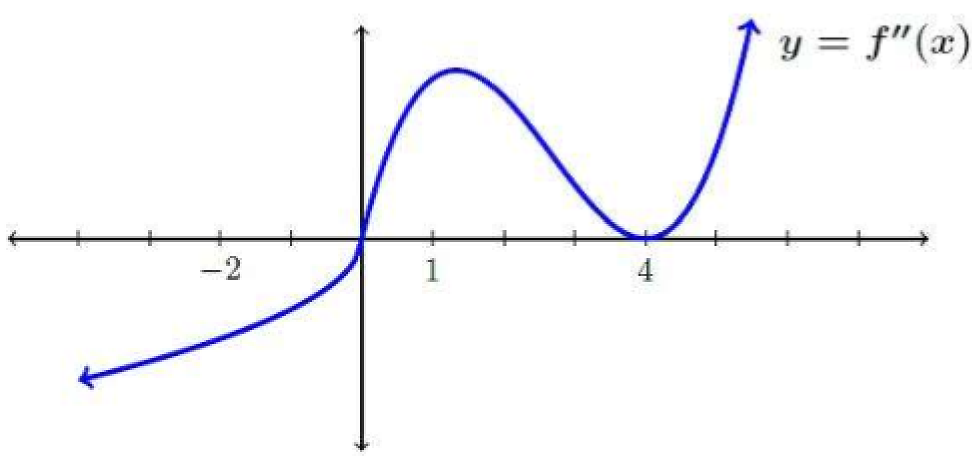
Free Response. Show all work for full credit.

1. (6 points) The radius of a circular oil spill is increasing at a constant rate of 1 meter per second. How fast is the area of the spill increasing when the radius is 20 meters?

2a. Find the point on the line $2x + y = 5$ that is closest to the point (-3,1).

2b. Evaluate $\lim_{x\to 0} \frac{x^3}{\sin x - x}$

2c. Consider the graph of $y = f''(x)$ below. If $y = f(x)$ has horizontal tangent lines at $x = -2$, 1,and 4, then where must $f(x)$ have a relative min?



3. If $f = \frac{(x-1)^2}{x^2+1}$, then $f''(x) = \frac{2(x^2-1)}{(x^2+1)^2}$ and $f''(x) = \frac{4x(3-x)^2}{(x^2+1)^3}$. Find the following:

Domain of f: ______________ Vertical asymptote(s): ___________

Horizontal asymptote(s): ________

Note: $\sqrt{3} \approx 1.7$, $f(-\sqrt{3}) \approx 1.9$, and $f(\sqrt{3}) \approx 0.1$

(a) f' and f'' number lines:

f'

f''

(b) x-intercept(s): $x =$ ___________;

y-intercept: $y =$ ___________ (write the exact value)

(c) relative minimum at $x =$ ___________;

relative maximum at $x =$ ___________

(d) inflection point(s) at $x =$ ___________

(e) Sketch the graph of $y = f(x)$ using the above information.