1. The area under the curve \( y = f(x) \) and above the \( x \)-axis on \([0,1]\) is represented by 
\[
\lim_{n \to \infty} \sum_{i=1}^{n} \sqrt[3]{\frac{1}{n}}. 
\]
Find the area.

2. Find min and max values of \( y = \sin^2 x + \sin x \) on \([0,2\pi]\)

3. Find critical numbers of \( y = x^2e^x \)

4. Evaluate \( \int_{0}^{1} \frac{x^2-1}{x^4+1} \, dx \)
5. Evaluate $\int \frac{3x^2}{x^3-4} \, dx$

6. Evaluate $\frac{d}{dx} \int_0^{x^2} \frac{t^3}{t^3+2} \, dt$

7. A soft drink dispenser pours a soft drink at the rate of $f(t) = \frac{20t}{1+2t^2}$ ml/sec where $t$ is the elapsed time in seconds. How much of the soft drink is dispensed in the first three seconds?

8. Consider the region between the curve $f(x) = 2 + 6x^2$ and the $x$-axis on the interval $[0,2]$. Calculate $R_4$, find the Riemann Sum approximation $R_n$, simplify it, and find the exact area by taking the limit of $R_n$ as $n \to \infty$. 
9. Sketch a graph of \( f(x) = \frac{x^2-9}{x^2-x-2} \)

10. Evaluate \( \lim_{x \to \infty} \left( \frac{x}{1-x} \right)^x \)

11. Let \( G(x) = \int_{-2}^{x} g(t) \, dt \) where \( g(t) = \frac{t(16-t^2)}{1+t^4} \). Which of the following statements is/are true?

P. \( G(x) \) is increasing on the interval \((-2,2)\)
Q. \( G(x) \) is an antiderivative of \( g(x) \)
R. \( G(x) \) has a local maximum at \( x = 4 \)

12. Find the value of the Mean Value Theorem implied by \( f(x) = x - \ln x \) on the interval \([1, e]\)

13. On which intervals is \( f(x) = \cos^2 x - 2 \sin x \) concave up on the interval \([0, 2\pi]\)?
1. (6 points) The radius of a circular oil spill is increasing at a constant rate of 1 meter per second. How fast is the area of the spill increasing when the radius is 20 meters?

2a. Find the point on the line $2x + y = 5$ that is closest to the point (-3, 1).

2b. Evaluate $\lim_{x \to 0} \frac{x^3}{\sin x - x}$

2c. Consider the graph of $y = f''(x)$ below. If $y = f(x)$ has horizontal tangent lines at $x = -2$, 1, and 4, then where must $f(x)$ have a relative min?
3. If \( f = \frac{(x-1)^2}{x^2+1} \), then \( f''(x) = \frac{2(x^2-1)}{(x^2+1)^3} \) and \( f''(x) = \frac{4x(3-x)^2}{(x^2+1)^3} \). Find the following:

Domain of \( f \): ______________ Vertical asymptote(s): __________

Horizontal asymptote(s): __________

Note: \( \sqrt{3} \approx 1.7 \), \( f(-\sqrt{3}) \approx 1.9 \), and \( f(\sqrt{3}) \approx 0.1 \)

(a) \( f' \) and \( f'' \) number lines:

(b) \( x \)-intercept(s): \( x = \) __________;

\( y \)-intercept: \( y = \) __________

(write the exact value)

(c) relative minimum at \( x = \) __________

relative maximum at \( x = \) __________

(d) inflection point(s) at \( x = \) __________

(e) Sketch the graph of \( y = f(x) \) using the above information.