

# Johann Sebastian Bach

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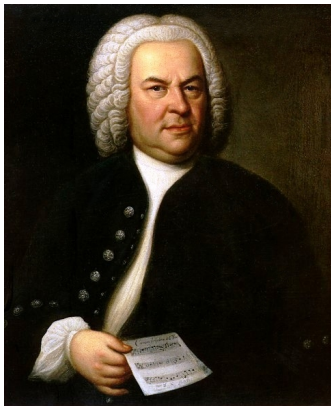
# Math and music go together

- Harmony: why certain combination of notes go together—ratios of their vibrational frequencies
- Subdivision of music into bars and beats
- Use of different rhythms
- Use of patterns and symmetry when writing music
- Different permutations and combinations of elements
- How small details form a bigger picture

Mathematicians like to

- Enumerate
- Classify
- Systematically work through all possibilities
- Look for patterns and symmetry in structures when trying to understand those structures

We see these traits in the music of Johann Sebastian Bach.



"A colossal syllable,  
one which makes composers  
tremble, brings performers  
to their knees, beatifies  
the Bach lover, and bores the  
daylights out of everyone else."

Leonard Bernstein,

"The Joy of Music"

# Patterns and Symmetry

Bach's music makes use of **counterpoint** where two or more melodies are woven together into a single piece of music. These melodies are independent, each functions as music in its own right. They should also complement each other.

If one melody has a fast pattern of notes another melody could be slow.

At some point they might then switch in some kind of dialogue .

If one melody is moving up in pitch and the other melody could be moving down in pitch.

# Four Corners of Bach's Musical World

**Chorale:** Protestant (Lutheran, for Bach) hymn in a harmonized arrangement, often for 4 voices. Ex. BWV 245 Ach Herr, lass dein lieb Engelein

**Choral Prelude:** a short piece of continuous texture from within, from time to time, a separate phrase appears, weaving in and out of the background chorale. Ex. BWV 244 Mattheus Passion

**Canon:** (Round) A device of strict imitation in which one melody is imitated at some time interval by one or more parts at the same or different pitch line of another. (Row row row your boat). Ex. BWV 988 Canone alla Quarta

**Fugue:** Grows out of the canon with tangents, episodes, and developments of increasing complexity. (Ex. 3 voice fugue from Toccata BWV 914 )

# Translations and Transformations

A melody/theme/phrase  $f(t)$  can be

- Staggered in time:  $f(t + C)$
- Staggered in pitch:  $f(t) + C$
- Inverted (played upside down):  $-f(t)$
- Retrograde (played backwards):  $f(-t)$
- Diminished (sped up):  $f(Ct)$ ,  $C > 1$
- Augmented (slowed down):  $f(t/C)$ ,  $C > 1$

More than one of these alterations can be used simultaneously.  
This can be done with 2 or more voices (6 in AMO ricercar a 6)

Gerubach's animation of BWV 1087 14 Canons

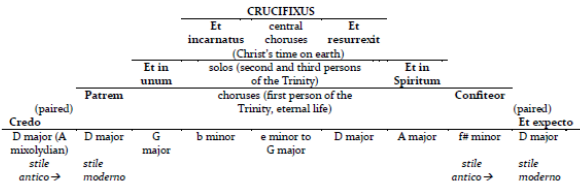
Math and the Musical Offering

Surface Topology in Bach Canons

# BWV 232 Mass in B Minor

Credo is the second of four "books" of the Mass in B minor

*Symbolum Nicenum: arch with Crucifixus as the keystone*



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Ex. Et Resurrexit Five choral parts enter in homophony then fugato.

<sup>1</sup> John Maclay, Bach Mass in B Minor Guide, 2013

# Passacaglia in C Minor BWV 582



Theme+ 20 variations.

Seven groups of three.

The fugue has 12 statements of the subject (21 backwards).



# Passacaglia in C Minor BWV 582

## Multiple levels of symmetry<sup>2</sup>

Chart No. 1

VAR.	1-2	3-5	6-8	9	10-11	12	13-15	16-18	19-20
Level A	<u>2 + 3</u>		<u>3 +1</u> L (3) J L		<u>1+1</u> - (4) -		<u>1 + 3</u> - J L (3) - J	<u>3 + 2</u>	
Level B	<u>5</u>		<u>4</u>		<u>2</u>		<u>4</u>	<u>5</u>	
Level C	<u>5</u>		<u>10</u>					<u>5</u>	

Expressed in another way:

Chart No. 2

1st Half

Var. \*1-2\* = 2

3-5 = 3

6-9 = 4

Var. \*10=1

\*11=1

(\*=paired variations)

2nd Half

Var. 12-15 = 4

16-18 = 3

\*19-20\* = 2

12

Ex. BWV 582

<sup>2</sup><https://www.jstor.org/stable/41639861>

# Goldberg Variations BWV 988

- Aria
- 30 Variations—10 sets of 3 (Virtuoso, Emotional, Rational (canon))
- Aria

Bach wanted to do as much as he could within this restrictive framework.

They all follow the same bass line which is 32 measures: 8 bars in G major, 8 bars in D major, 8 bars in E minor 8 bars in G major

The number 32 is important and shows up in the internal divisions of the piece. Some of his variations are 3 beats to a measure and some are 2. There is almost an exact ratio of 3:2.

# Goldberg Variations BWV 988

Every third variation is a strict canon.

- Variation 3: first canon is a unison canon (like Frere Jaques—second voice starts on the same starting Pitch as the first).
- Variation 6: A canon at the second (second voice starts on an interval of 2 i.e. a note higher than the first).
- Variation 9: A canon at the third, etc.

The other variations take many different forms:

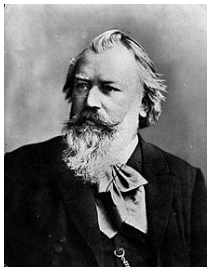
- Fugue
- Aria
- French Overture
- Dances

# Chaconne

from Partita no. 2 in D minor, BWV 1004 (1720)

Four-note theme + sixty-ish variations, each four measures long.  
(Ex. arr. Ferruccio Busoni)

"On one stave, for a small instrument, the man writes a whole world of the deepest thoughts and most powerful feelings. If I imagined that I could have created, even conceived the piece, I am quite certain that the excess of excitement and earth-shattering experience would have driven me out of my mind."



*Johannes Brahms*

# Two years later

French Suite # 5 BWV 816 G Major

(Ex. Gigue)

AABB Form

A section: modulation leads to key of D (dominant)

B section: inversion of the theme (upside down), starting in D,  
modulation back to G

- BWV 831 French Overture
- Cadenza to Brandenburg 5 BWV 1050
- English Suite #2 in A minor, Prelude, BWV 807
- Keyboard Concerto #1 BWV 1052
- Nun Seid Ihr wohl gerochen, Christmas Oratorio BWV 249
- Sinfonia, Cantata BWV 29
- Cum Sanctu Spiritu, Mass in B, BWV 232

# Bernard Chazelle on Bach

To reduce the man's genius to the vastness of his musical brain would be a mistake.

My fanhood is shamelessly unintellectual. **I love Bach because his music is the most formidable elation machine ever engineered.**

Analysis is optional. The thrill is not. So forget the cerebral razzle-dazzle. The music is corporeal, sensuous, and intoxicating. Bach, the most human of all composers, gets to your soul through your body.

New gym exercises acquaint us with muscles we didn't know our bodies even possessed. Likewise, Bach's music awakens in us a multitude of sensibilities that would lie dormant without it. It reminds us of the aesthetic virtues that live, often hidden, inside each one of us.

# Jeremy Denk on Bach

I often talk about Bach as a great humanist, as having an empathy for the whole range of human emotion. (Rather than the cerebral, fugal stereotype.) I love the way his music seems to look down on the whole human deal, but not condescendingly, with a kind of benevolent understanding.

He does not look down bitterly (like Shostakovich), saying look at this terrible empty comedy of human emotion.

Nor is he himself the emoter, like Beethoven; but he is not distanced, either. He has hit a sweet spot.

Perhaps the most serious complaint you could make about Bach is that he has every quality of humanity except imperfection.





# Emil Cioran on Bach

Bach's music is the only argument proving the creation of the Universe cannot be regarded as a complete failure.

