Born's rule: a look behind the scenes

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Abstract

The Born rule presents the measurement process as an atomic action of a projection of the state on a subspace of the Hilbert space of quantum states. Here we take a detailed look at what happens in a real, as opposed to a thought, double-slit experiment. There are two roles for statistics and probabilities in quantum mechanics: (1) statistics is needed to manage the data from observations of quantum phenomena; and (2) the theory to explain and organize quantum phenomena is a statistical theory.

This is note # 2 in a series of notes to untangle quantum mechanics for a general audience and experts alike.

URL https://people.clas.ufl.edu/deumens/files/pap-behind.pdf

The June and July 2022 issues of Physics Today each had a Quick Study about the measurement problem in quantum mechanics (QM). The first article by David Mermin [3] gives the position that there is no problem, which is the extreme logical endpoint of a century-long evolution of the Copenhagen view. The second article by Sean Carroll [1] points out that there are questions about the measurement process that require a careful answer. He gives an overview of the most widely respected alternatives, and finishes with the expectation that some experiment, however challenging, may bring clarity among the alternatives.

Classical physical systems have properties that are described as a finite set of numbers (mass, charge, pressure, color, etc.). It is assumed that for every system there exist smaller, more delicate, systems that can be used as probes to explore any properties in such a way that the interactions are subtle and can be neglected. In other words, one can measure objects without having to describe the measurement as a physical process subject to the same laws that govern the dynamics of the systems being observed.

QM is the theory that claims validity down to the smallest scales of known physical phenomena. And, thus far, it has delivered on that claim. That frontier characteristic

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implies that there is no finer, smaller, subtler system available to measure the quantum systems. This note analyzes in depth the dynamics of a double-slit experiment to show the complexity of the measurement process. It will show that there is a lot going on in a measurement process.

The double slit experiment We choose to analyze the double-slit experiment because of its historical role and its ubiquitous discussion in textbooks. In 1989, Tonomura, Endo, Matsuda, Kawasaki, and Ezawa [4] did a beautiful and detailed version of the experiment with electrons that clearly shows how the interference pattern is built up from many individual flashes. The experiment is configured around an electron microscope, which provides the controllable beam of electrons being sent to the detector. A full description [2] is beyond the scope of this note. The essential point is that the detector of the electron impact consists of fluorescent molecules, which emit photons that are captured by optical fibers, which then eventually lead to a recording in the memory of a computer.

Typically, fluorescent molecules consist of about 30 atoms and they have a diameter of 10 nm, which gives them a cross section surface of 80 nm^2. The film has 10^{12} molecules, each of which can send one photon when excited by a collision with the incoming electron. Since one 5,000 Volt electron can produce 500 photons, it must interact with 500 fluorescent molecules to produce the 500 photons. We call the collection of a group of 500 fluorescent molecules a mini detector. There are then $2 \times 10^9 = 10^{12}/500$ mini detectors in the fluorescent film of the detector screen, each with a surface area of 500 x 80 nm^2 = 0.04 μm^2. An optical fiber has a cross section of 80 μm^2. That means that one optical fiber captures the 500 photons from 80 μm^2 / 0.04 μm^2 = 2,000 mini detectors. Each mini detector has $500 \times 30 = 15,000$ atomic degrees of freedom. The state of the mini detector depends on these 15,000 variables. The statistics of 15,000 variables has a standard deviation $\sqrt{15,000} = 122$ times smaller than that of the original atomic variables. It follows that the quantum mechanical fuzziness from possible superpositions in the atomic variables, that describe whether the mini detector has fired off 500 photons or not, is now like a sharp classical variable with a precise value of yes or no.

Two roles for statistics Starting from the, now sizeable, collection of interesting experiments demonstrating the various quantum phenomena, it is possible to develop an intuition and a set of concepts and practical rules to think about, analyze, and predict the phenomena. In short, live with them. Even though the quantum phenomena are not part of our everyday life, it is possible to deal with them, when they come up, in a way similar to the way you handle everyday phenomena, usually described as “clasical” in physics. One very important difference between classical and quantum phenomena is that quantum phenomena invariably involve probabilities and statistics. In contrast, it is possible in classical phenomena to find simplified versions that can be handled without probabilites and with predictable precision.

Therefore, to describe the complex dynamics of the interaction of the electron with the vast number of mini detectors, we need a probabilistic approach, like in statistical mechanics. Because of the probabilistic nature of quantum phenomena, it is not possible to find a determinsitic mechanics, like Newtonian mechanics, that explains the quantum phenomena. But note that that does not mean that the statistical mechanics does not have an underlying deterministic dynamics; it means that we have to formulate the theory as a statistical mechanics to account for the probabilistic nature inherent in the quantum phenomena. The structure of statistical mechanics for quantum phenomena is the same as that of classical statistical mechanics for describing classical phenomena of complex systems that cannot be simplified to versions with predictable precision. We need to consider a probability distribution of the states of the electrons flying in and of the states of all the mini detectors in the fluorescent film. So the *quantum theory needs statistics*, as does the description of the quantum phenomena, but the nature of the statistics is different. The probability enters in two ways: (1) in the initial conditions of the systems, just as in classical statistical mechanics, and (2) in the boundary conditions where uncontrolled external forces influence the deterministic dynamics given by the Schrödinger equation, like the processes described in classical stochastic mechanics.

The statistics for the description is the statistics associated with processing data from observations.
The statistics in quantum theory concentrates on the initial conditions and boundary conditions.

Statistical state of the electron beam – In the electron microscope the state of the electrons as they are sent out from the electron gun is quite precisely determined. That means the state as a vector in Hilbert space, or equivalently as a wavefunction, is pretty similar, even if not identical, for all electrons sent down to the double slit. This is the wavefunction that is used in all theoretical discussions. The statistical description then involves a probability distribution on wavefunctions for the electron that is sharply peaked around that one well-known wavefunction with a small standard deviation or error.

Statistical state of the fluorescent film – The statistical distribution of the film, however, is not like that at all. We have seen that the film contains 2×10^9 mini detectors, each consisting of 500 fluorescent molecules with 30 atoms in them. These atoms and molecules are wiggling around at room temperature, pushing against each other. They do not freely move around, because the film is a solid and not a liquid, but these atoms and molecules are not sitting still. We do not know what the precise states (vectors or wavefunctions) of the molecules are. We do know something about them: We do know that the atoms are all bound together into the structure of the fluorescent molecules, these bonds are not breaking any time during the experiment. That means we need to think of the film in a statistical way and describe it with a probability distribution of states (vectors or wavefunctions). And for the film, this probability distribution is not narrowly focused on one specific state for each molecule, but rather gives a broad spectrum of possible states for the molecules. All molecules are in different states and so are, as a consequence, the mini detectors, which consist of 500 molecules.

Evolution of statistical states – To determine what will happen during the experiment, we need to treat the Shrödinegr equation (SE) in a statistical way. How to do this is known from statistical (classical) mechanics. The method requires a very technical discussion, and we will not get into the mathematical details. The way it works is by looking at the dynamics of all the wavefunctions of the incoming electrons, which we know are all similar, so we can use the one most probable wavefunction. Then we use the SE to describe the interaction with all the possible and probable states of the mini detectors, which, as we just saw, can vary greatly in the estimated probability distribution of the fluorescent film, so we must consider all these options. Then we compute the average with the probability distribution to get the observed result.

Let us describe in some detail what can happen. Three possibilities stand out.

1. In some combinations of electron-state and mini-detector-state the SE predicts that nothing noticeable happens. The electron jiggles the molecules some, but no fluorescent molecules get put into an excited state with subsequent emission of a photon. So, nothing is detectable.
2. In some cases, the interaction of the electron with a mini detector excites a number of fluorescent molecules and they each emit one photon. But the total number is small compared to the 500 possible photons that can be created with the energy in the electron. Nothing detectable happens, the signal from a few photons is too weak.
3. In some mini detectors, the states (wavefunctions) of all, or most, of the fluorescent molecules are such that the interaction as described by the SE has the electron put all of them in an excited state that then decays with the emission of a photon each. That produces 500 photons taking all the energy from the 5,000 Volt impinging electron.

In the case that the probability distribution of the wavefunction of the fluorescent film has a high probability for finding large numbers of mini detectors, and fluorescent molecules in them, in situation (3), then there will be a flash of 500 photons with high probability. That event will trigger a detection of the flash in the computer memory and a dot will be displayed on the monitor.

Every measurement in QM has an underlying physical quantum process as complicated as this one. The generic complexity of any and all the measurement processes is a consequence of the large number of quantum degrees of freedom involved, which makes solving the SE very challenging. Born's rule has been very useful for all experiments carried out during the first century of QM.

References

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