Closer look at superposition and entanglement

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Abstract

Most discussions about superposition and entanglement happen in a data-science context. In the mathematical framework of quantum mechanics, both are linear combinations in the Hilbert space of states.

This note # 6 in a series of notes to untangle quantum mechanics for general audience and experts alike.

URL https://people.clas.ufl.edu/deumens/files/pap-entangle.pdf

The word "entanglement" is used in the context of quantum information science to describe a special correlation between observations on quantum systems that are in a coherent quantum state. Entanglement has been called the most important feature of quantum mechanics [3] and is crucial to the functioning of quantum computers [4].

Examples of such quantum states are the state of a pair of photons created by a suitable decay process of a molecular or atomic excited state such that the photons have equal polarization. Or, the state of a systems of two electrons ejected from a bound state with total spin zero, producing two electrons with spins that are oriented opposite of each other. Such states are known in the literature as "Einstein-Podolsky-Rosen (EPR) states," because the paper published by these three in 1935 [2] introduced them, or as "Bell states" because of the theorem Bell published in 1964 [1] while working on the EPR problem. The observations of the two photons or electrons in such an entangled state give perfectly correlated results for the polization and spin, respectively, when these are measured in the same direction for both systems. These results are obtained even though the measurements in quantum mechanics are probabilistic. The most remarkable aspect, that was the basis for the argument in the EPR paper, is that this phenomenon occurs no matter how far the two systems, photons and electrons, respectively, may have traveled away from each other.

While entanglement looks quite paradoxical looking at the observations, in the mathematical formalism of quantum mechanics, which describes quantum states as vector in a

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Hilbert space, an entangled state is described as a linear superposition of two other states, which is just the natural linease operation that is basic in Hilbert spaces. There is nothing remarkable about it when viewed that way.

We recognize two flavors of superposition. For a single system, or more precisely and system with a single degree of freedom, a superposition is the linear combination of two states. For example, a single qubit has two basis states most often denoted as $|0\rangle$ and $|1\rangle$ and a general qubit state is then the superposition $\alpha|0\rangle + \beta|1\rangle$. Compared to classical bits that are either 0 or 1, and never some mixture or combination of these two states, this superposition state, while new to quantum mechanics, is not that strange. It fits with the probabilistic nature of quantum phenomena and it means a measurement of the qubit in that state with get $|\alpha|^2$ % of the time the result 0 and $|\beta|^2$ % of the time the result will be 1.

For a system with two degress of freedom or for two coupled systems, the superposition of states gives rise the the observations of entanglement between the results from the separate degrees of freedom or systems. And then it does look unexpected, remarkable, and paradoxical to our classically trained intuition.

References

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