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International Trade and Volume Patterns under Quasilinear Preferences

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Abstract

We formally analyze the pattern and volume of trade by embedding quasilinear preferences in the standard perfectly competitive, two-factor, two-good, two-country trade model. Quasilinear preferences deliver a natural partition of the two goods into a luxury and a necessity, and preserve the validity of the Heckscher–Ohlin and Heckscher–Ohlin–Vanek theorems. In addition, the predicted factor content of trade under quasilinear preferences is smaller (larger) than the predicted factor content of trade under homothetic preferences if and only if the luxury good is capital (labor) intensive. This result offers a novel explanation for the "missing-trade" mystery.

1. Introduction

The assumption of homothetic preferences has been used routinely by international economists and has served as a useful simplification in the analysis of supply-side determinants of the structure of international trade.[1] This assumption has been questioned along two broadly defined dimensions. First, convincing empirical evidence has been accumulated in support of nonhomothetic preferences.[2] More recently, nonhomothetic preferences have been recognized as a possible determinant and explanation for the deviation between the observed and predicted factor content of trade, the so-called "missing-trade" mystery (Trefler, 1995).[3] Second, a small but growing body of theoretical literature has employed various forms of nonhomothetic preferences to address a variety of international trade issues.[4]

Quasilinear preferences represent a particular type of nonhomothetic preferences. In the case of two products, they are modeled by assuming that consumer utility is a linear function of the quantity of one good plus a concave function of the quantity of the other good. It would not be an exaggeration to state that general-equilibrium theorists view quasilinear preferences as a simplifying assumption that either justifies the use of partial-equilibrium analysis, or allows the researcher to focus on product markets by abstracting from potentially complicated factor-market-based effects. For example, Feenstra (2004, ch. 7) justifies the use of a partial-equilibrium approach to commercial policy by combining quasilinear preferences with a Ricardian structure of production; Grossman and Helpman (1994) assume quasilinear preferences and sector-specific factors of production to analyze the role of campaign contributions on protection; Melitz and Ottaviano (2008) add quasilinear preferences to a single factor model to

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analyze the effects of market size on industry productivity and markups; and Ottaviano et al. (2002) combine quasilinear preferences with sector-specific factors in a model of trade and agglomeration.

Our research will argue that quasilinear preferences behave reasonably well in general-equilibrium settings and exhibit several desirable properties. These properties place them at or above the class of analytical tools which includes homothetic and Stone–Geary preferences. Our argument starts by pointing out that typical general-equilibrium models with quasilinear preferences assume that the good whose quantity enters linearly in the consumer utility function is also produced by a *single* factor of production under constant returns to scale. This assumption, which has been used by Feenstra (2004, ch. 7), Grossman and Helpman (1994), and Melitz and Ottaviano (2008) among many others, neutralizes the general-equilibrium link between product and factor markets.

In the present paper, we assume that the good whose quantity enters linearly in the utility function is produced with two factors of production under constant returns to scale and perfect competition. This modification allows us to replace the assumption of homothetic tastes with that of quasilinear preferences in the traditional two-country, two-factor, two-good trade model without any additional supply-side restrictions. We then address the following questions. What are the effects of quasilinear preferences on the joint determination of per capita income and the structure of commodity trade? Do standard (or modified versions of the) Heckscher–Ohlin (HO) and Heckscher–Ohlin–Vanek (HOV) theorems apply to a world of quasilinear preferences? Does the magnitude of the factor content of trade remain intact?[5] If not, can quasilinear preferences provide a possible theoretical explanation for the "missing-trade" paradox?

Our analysis generates several surprising and, according to our view, interesting findings that are summarized below. Quasilinear preferences offer a natural classification of the two goods into a luxury and a necessity. The former corresponds to the good whose quantity enters linearly in the utility function: this good is characterized by a constant marginal utility; its consumption expenditure share increases in per capita income; and its quantity demanded goes to zero as per capita income approaches a lower positive bound. The necessity good exhibits diminishing marginal utility: its quantity demanded is independent of per capita income; its consumption expenditure is a convex and declining function of consumer income; and its quantity demanded is always positive as long as consumer income is positive.

Lemma 1 establishes that the country with a higher per capita supply of the luxury (income-sensitive) good exports that good. This result, which does not hold for the case of Stone–Geary preferences, implies that quasilinear preferences preserve the role of factor abundance as the sole determinant of the pattern of trade: a country exports the good that uses intensively its abundant factor of production and imports the good which uses intensively its scarce factor of production (Theorem 1). The HOV theorem applies as well: under quasilinear preferences, a country exports factor services produced by its abundant factor of production and imports factor services produced by its scarce factor of production (Theorem 2). This property highlights a novel difference between quasilinear and Stone–Geary preferences. Under the latter, as shown by Hunter and Markusen (1988, p. 108), the HO and HOV theorems can break down and a country could in principle export the good that uses intensively its scarce factor of production.

Under the reasonable assumption that the luxury good is capital intensive, the factor content of trade under homothetic preferences is larger than the factor content of trade under quasilinear preferences. The opposite holds if the luxury good is labor intensive

(Theorem 3). Therefore quasilinear preferences offer a novel theoretical explanation for the "missing-trade" mystery (Trefler, 1995).

2. Properties of Quasilinear Preferences

Consider now a two-good (clothing and food) and two-country (Home and Foreign) world economy populated by identical consumers. The preferences of each consumer are nonhomothetic and described by the following quasilinear utility function:[6]

$$U(q_0, q_1) = q_0 + u(q_1), \quad u'(\cdot) > 0, \quad u''(\cdot) < 0, \tag{1}$$

where q_0 is the quantity of clothing and q_1 is the quantity of food consumed. Let I denote per capita income, set the price of clothing equal to unity ($p_0 = 1$) by choice of the *numéraire*, and denote with $p = p_1/p_0 = p_1$ the relative price of food. Maximization of (1) subject to the standard budget constraint $I = pq_1 + q_0$, generates the first-order condition $p = u'(q_1)$. Inverting this condition yields the following per capita demand function for food:

$$q_1 = u'^{-1}(p) \equiv D(p), \quad D'(\cdot) < 0. \tag{2}$$

Substituting equation (2) into the consumer budget constraint generates the demand function for clothing:

$$q_0 = I - pD(p). \tag{3}$$

Equations (2) and (3) can be used to illustrate the basic properties of quasilinear preferences. First, there is a threshold level of income, defined by setting the right-hand side of equation (3) equal to zero (i.e. $I_{\min} = pD(p)$) below which the consumer does not buy any clothing (i.e. $q_0 = 0$) and spends all income on food. In order to avoid corner solutions, we assume that each consumer has sufficient income to consume both goods. In other words, we suppose that the parameters of the model are such that the quantity of clothing, q_0, is strictly positive in equilibrium. Second, any change in consumer income is reflected only on the demand for clothing, which increases linearly in per capita income.[7]

Third, each consumer spends a fixed amount of money on food equal to $pD(p)$ and the rest of her income is allocated to clothing. Thus the consumption expenditure share of food is given by $pD(p)/I$, and therefore it is a declining and convex function of per capita income. In contrast, the consumption expenditure share of clothing, which is given by $1 - pD(p)/I$, is an increasing and concave function of per capita income. Needless to say, this property enjoys empirical support as well.[8] These properties suggest that quasilinear preferences offer a natural partition of the two goods into a "luxury" good (clothing) with constant marginal utility and a "necessity" good which exhibits diminishing marginal utility (food).

Fourth, quasilinear preferences yield a relative demand (for food), q_1/q_0, which declines in per capita income I. More specifically, equations (2) and (3) imply that the income consumption path is a vertical line in the q_1- and q_0-space intersecting the q_1-axis at $q_1 = u'^{-1}(p)$. Fifth, observe that the aggregate demand for each good is directly proportional to the size of the market measured by number of consumers. The last two properties mean that the aggregate demand for each good is independent of the distribution of income among factor owners. As a result, quasilinear preferences generate trade patterns that are independent of the distribution of income within each country.

In what follows, an asterisk (*) will be used to identify variables and functions of Foreign. Therefore, let L and L^* denote the number of Home and Foreign consumers; and let Y_1 and Y_1^* denote the Home and Foreign supplies of food. Under free trade, the market-clearing condition requires that the global demand for food equals its global supply:

$$LD(p)+L^*D(p)=Y_1+Y_1^*, \tag{4}$$

which can be rewritten as:

$$L\left[D(p)-\frac{Y_1}{L}\right]+L^*\left[D(p)-\frac{Y_1^*}{L^*}\right]=0, \tag{5}$$

and leads immediately to the following criterion that determines the pattern of trade:

LEMMA 1. *The country with the higher per capita supply of food exports food and imports clothing* (*i.e. Home exports good* q_1 *if and only if* $Y_1/L > Y_1^*/L^*$).

The proof will be found in the Appendix.

The economic intuition behind Lemma 1 is as follows. Although income per capita might differ between Home and Foreign, per capita demand for food depends only on its price and thus is independent of consumer per capita income thanks to the quasi-linear functional form of the utility function. Equation (5) implies that Home's autarky price of food, p_A, is given by $D(p_A) = Y_1/L$ and Foreign's autarky price of food, p_A^*, is given by $D(p_A^*) = Y_1^*/L^*$. Thus the autarky price of food is lower in the country with the higher per capita supply of food. Therefore, the country with a higher per capita supply (and lower autarky relative price) of food exports this good and imports clothing.

Lemma 1 does not apply to the case of Stone–Geary preferences under which the demand of every good (as opposed to only one good) depends on income per capita, minimum consumption requirements, and prices of all goods.[9] Notice, though, that Lemma 1 does not relate directly factor-abundance differences to the trade pattern. The following section addresses this task.

3. Trade Patterns

Assume now that each of the two goods is produced with capital and labor under constant returns to scale, perfect competition, and same technology across the two countries. Each worker/consumer is endowed with one unit of labor which is supplied to firms producing the two final goods. In addition, assume that each country remains incompletely specialized after trade.

Denote with K (K^*) the fixed endowment of capital at Home (Foreign), with w (w^*) Home (Foreign) wage of labor, and with r (r^*) Home (Foreign) rental of capital. The production side of the world economy is then described by the following equations:

$$c_0(r,w)=1, \tag{6}$$

$$c_1(r,w)=p, \tag{7}$$

where $c_i(r, w)$, $i = 0, 1$ is the unit–cost function for good i which is concave and homogeneous of degree one in both arguments. Equations (6) and (7) are simply the zero-profit conditions for clothing and food, respectively. Under free trade and nonspecialization in production, the relative price of food p is common in both countries and therefore equations (6) and (7) deliver factor–price equalization (i.e. $w = w^*$ and $r = r^*$).

Let w and r be the common global factor prices, $a_{Ki}(r, w) = \partial c_i(r, w)/\partial r$ be the per unit capital requirement; and $a_{Li}(r, w) = \partial c_i(r, w)/\partial w$ be the per unit labor requirement in the production of good i. Then the full-employment conditions at Home are:

$$a_{K0}(r,w)Y_0 + a_{K1}(r,w)Y_1 = K, \tag{8}$$

$$a_{L0}(r,w)Y_0 + a_{L1}(r,w)Y_1 = L. \tag{9}$$

Similarly, the full-employment conditions at Foreign are:

$$a_{K0}(r,w)Y_0^* + a_{K1}(r,w)Y_1^* = K^*, \tag{10}$$

$$a_{L0}(r,w)Y_0^* + a_{L1}(r,w)Y_1^* = L^*, \tag{11}$$

The system of six equations (6) through (11) determines the equilibrium values of six endogenous variables ($r, w, Y_0, Y_1, Y_0^*, Y_1^*$) for any given value of the relative price p. The global market-clearing condition for food (4) provides the final equation of the system.

As a reminder, let us analyze first the case of a small open economy, say Home, where the relative price p is treated as an exogenous parameter and the economy is described by equations (6), (7), (8), and (9). In this case, the structure of demand does not play any role in the determination of trade patterns. Equations (6) and (7) determine the values of the two factor prices w and r. Totally differentiating these two zero-profit conditions leads to the standard Stolper–Samuelson theorem: an increase in the price of a commodity raises the relative price of the factor that is used intensively in its production. Equations (8) and (9) generate the Rybczynski theorem: an increase in the supply of a factor of production raises the relative supply of the good that uses that factor intensively in its production.

We next establish the robustness of the HO theorem under quasilinear preferences. In view of Lemma 1, one could relate per capita output of food to national factor endowments. The two Home full-employment conditions yield:

$$\frac{Y_1}{L} = \frac{a_{L0}(r,w)\dfrac{K}{L} - a_{K0}(r,w)}{a_{K1}(r,w)a_{L0}(r,w) - a_{K0}(r,w)a_{L1}(r,w)}. \tag{12}$$

Similarly, the two Foreign full-employment conditions generate

$$\frac{Y_1^*}{L^*} = \frac{a_{L0}(r,w)\dfrac{K^*}{L^*} - a_{K0}(r,w)}{a_{K1}(r,w)a_{L0}(r,w) - a_{K0}(r,w)a_{L1}(r,w)}. \tag{13}$$

Comparison of equations (12) and (13) establishes the desired result, namely,

$$\operatorname{sign}\left\{\frac{Y_1}{L} - \frac{Y_1^*}{L^*}\right\} = \operatorname{sign}\left\{\frac{K}{L} - \frac{K^*}{L^*}\right\}, \tag{14}$$

if and only if food is capital intensive (i.e. $a_{K1}(r, w)a_{L0}(r, w) - a_{K0}(r, w)a_{L1}(r, w) > 0$). This result and Lemma 1 lead to

THEOREM 1. *(Heckscher–Ohlin): Under quasilinear preferences, perfect competition, and incomplete specialization in production, a country exports the good that uses intensively its abundant factor of production and imports the good that uses intensively its scarce factor of production.*

The proof of Theorem 1 relies on Lemma 1 and the per capita version of the Rybczynski theorem. The latter implies, for example, that an increase in capital abundance (measured by an economy's capital/labor ratio) raises per capita production of the capital-intensive good and reduces per capita production of the labor-intensive good. Consequently, the capital-abundant country generates a higher per capita supply (and a lower autarky price) of food, if and only if food is the capital-intensive good. Therefore the capital-abundant country exports the capital-intensive good. Theorem 1 differentiates quasilinear preferences from the linear expenditure demand system, which corresponds to Stone–Geary preferences. The latter imposes a "strong" form of nonhomotheticity and generates deviations from the traditional HO theorem that can reverse the trade pattern: the capital-abundant country could export the labor-intensive good.

We now turn our attention to the validity of the HOV theorem which is the mirror image of the HO theorem in our setting. Without loss of generality consider Home's economy and denote with $F_K \equiv a_{K1}(Y_1 - Lq_1) + a_{K0}(Y_0 - Lq_0)$ the amount of capital embedded in net exports, where $Y_1 - Lq_1$ and $Y_0 - Lq_0$ are net exports of food and clothing, respectively. Similarly, Home's labor content of net exports is defined by $F_L \equiv a_{L1}(Y_1 - Lq_1) + a_{L0}(Y_0 - Lq_0)$. The Appendix derives the following expressions for Home's factor content of trade:

$$F_K = wa_{L0}(K - \sigma K^W), \tag{15}$$

$$F_L = -ra_{L0}(K - \sigma K^W), \tag{16}$$

where $K^W \equiv K + K^*$ is the world capital endowment, $L^W \equiv L + L^*$ is the world labor endowment, and $\sigma \equiv L/(L + L^*)$ is Home's share of world population. The term in parenthesis in equations (15) and (16) is positive if and only if Home is capital abundant (i.e. $K/L > K^W/L^W$) and therefore we obtain

THEOREM 2. *(Heckscher–Ohlin–Vanek): A country exports factor services produced by its abundant factor of production and imports factor services produced by its scarce factor of production.*

The intuition behind Theorem 2 is as follows. The capital-abundant country produces capital services cheaper in autarky and exports these services to the capital-scarce country. Under quasilinear preferences, per capita income differences across countries are not sufficient to reverse the traditional trade pattern, and therefore the Stolper–Samuelson, Rybczynski, HO, and HOV theorems all hold.

4. Factor Content of Trade

Under quasilinear preferences, per capita demand of clothing and the relative demand of food depend on per capita income. This dependence introduces a novel difference between the factor content of trade under quasilinear preferences and the factor content of trade and under homothetic preferences. The present section explores this difference and relates it to the "missing-trade" mystery.

The analysis can be readily generalized to the case of many factors and many countries; but for clarity of exposition and notational consistency, we focus on the two-good, two-country, two-factor trade model. Consider the case of two global economies identical in all respects except the structure of preferences: one is populated by consumers with homothetic preferences and the other is populated by consumers with quasilinear preference captured by equation (1). Theorems 1 and 2 assert that in both global economies the labor-abundant country exports the labor-intensive good and labor services. Which of the two global economies will be characterized by a lower volume of trade as measured by the factor content of trade? What conditions generate lower trade volume under quasilinear preferences?

Following Leamer (1980), one can derive the following HOV expression for the factor content of international trade under the assumption of homothetic tastes. Without loss of generality denote with F_K^H and F_L^H the capital and labor content of Home's net exports. In a two-country and two-factor world, it suffices to focus on Home only. Balanced trade implies that Home's factor content of trade is identical (abstracting from sign considerations) to Foreign's factor content of trade. Home's factor content of trade is given by

$$F_K^H = K - sK^W, \tag{17}$$

$$F_L^H = L - sL^W, \tag{18}$$

where $s = (rK + wL)/(rK^W + wL^W)$ is Home's share of world expenditure (world GDP); $K^W = K + K^*$ is the world capital endowment; and $L^W = L + L^*$ is the world labor endowment. According to equation (17) Home's supply of capital services equals its capital endowment K, whereas its demand for capital services is sK^W, and therefore the capital content of net exports, F_K^H, equals Home's net excess supply of capital. Equation (17) states that if Home is abundant in capital (i.e. $s < K/K^W$) the value of F_K^H is positive and consequently it exports capital services to Foreign. In this case it can be readily shown that $s > L/L^W$. Thus the value of F_L^H is negative, implying that Home imports labor services from Foreign.

Following the notation of the previous section, denote with F_K and F_L the capital and labor content of Home's net exports in a global economy populated by consumers with quasilinear preferences. The factor content of trade in this economy is given by equations (15) and (16). The Appendix derives the following expressions for the difference between the factor content of net exports under quasilinear and homothetic preferences:

$$F_K - F_K^H = w\frac{ra_{L0}LL^W}{(rK^W + wL^W)}\left(\frac{K}{L} - \frac{K^W}{L^W}\right)\left(\frac{K^W}{L^W} - \frac{a_{K0}}{a_{L0}}\right), \tag{19}$$

$$F_L - F_L^H = -r\frac{ra_{L0}LL^W}{(rK^W + wL^W)}\left(\frac{K}{L} - \frac{K^W}{L^W}\right)\left(\frac{K^W}{L^W} - \frac{a_{K0}}{a_{L0}}\right). \tag{20}$$

Equations (19) and (20) reveal the role of factor abundance and factor intensities as the two principal determinants of the bias in the factor-content of trade between the two systems of preferences. To see this, consider the case where Home is the capital-abundant (advanced) country (i.e. $(K/L) > (K^W/L^W)$) engaging in free trade with a labor-abundant (developing) country. Suppose also that clothing is the capital-intensive good (i.e. $(a_{K0}/a_{L0}) > (K^W/L^W)$). These conditions imply that the right-hand side of equation (19) is negative and, therefore, $F_K^H > F_K$. In addition, Theorems 1 and

2 imply that, in this case, Home exports capital services embodied in clothing and imports labor services embodied in food independently of whether preferences are quasilinear or homothetic. This means that $F_K > 0$, $F_K^H > 0$ in equation (19) and $|F_K^H| > |F_K|$: the capital content of Home exports under homothetic preferences exceeds the capital content of Home exports under quasilinear preferences.

Similar considerations apply to the labor content of Home imports. If Home is capital-abundant and clothing the capital-intensive good, the right-hand side of equation (20) is positive; that is, $F_L - F_L^H > 0$. However, in this case, Home imports labor services and both terms of the left-hand side of (20) are negative ($0 > F_L > F_L^H$). This property implies that $|F_L^H| > |F_L|$: the labor content of Home imports under homothetic preferences exceeds the labor content of Home imports under quasilinear preferences. In contrast, if clothing is labor intensive, then quasilinear preferences generate a higher factor content of trade compared to homothetic preferences. Balanced trade between Home and Foreign means that these results hold for Foreign as well. This analysis is summarized by the following theorem:

THEOREM 3. *The volume of trade in factor services under quasilinear preferences is smaller (larger) than the trade volume under homothetic preferences if and only if clothing (the luxury good) is capital (labor) intensive.*

The economic intuition behind Theorem 3 is as follows. Consider first two countries with identical factor abundance. In this case none of the two countries engages in trade because each country has the same per capita income (i.e. $I = I^* = rk^W + w$, where $k^W = K^W/L^W$ is the world capital abundance), and the same per capita production of each good (see Lemma 1). Starting at the no-trade equilibrium, consider now a marginal redistribution of capital from Foreign to Home. This redistribution of capital does not affect the world factor endowments but transforms Home into the capital-abundant country. As long as both countries are incompletely specialized, this change generates an "excess" supply of capital at Home which is exported to Foreign in exchange for Foreign's labor services. The opposite holds for Foreign.

Without loss of generality, one can focus on the effects of this redistribution on Home's economy. An increase in Home's capital endowment generates supply-and-demand effects. The supply effect is delivered by the Rybczynski theorem and raises Home's supply of the capital-intensive good which is exported to Foreign *ceteris paribus*. The demand effect is associated with an increase in Home's per capita income relative to the no-trade equilibrium (i.e. $I = rk + w > I^* = rk^* + w$, where $k = K/L > k^* = K^*/L^*$). An increase in Home's per capita income does not affect the aggregate demand for food, $Lq_1 = LD(p)$, but increases the demand for clothing, $L_{q_2} = L[rk + w - pD(p)]$. Thus, if clothing is capital intensive, the demand effect tends to reduce the "excess" supply of Home's capital increase and generates lower trade volume. In contrast, if clothing is labor intensive the demand effect amplifies the "excess" supply of Home's capital endowment and augments the absolute value of the factor content of global trade. Under homothetic tastes there is no demand effect since aggregate demand depends only on relative prices.

The difference between the factor content of trade under quasilinear and homothetic preferences can be readily translated into cross-country differences in per capita income. To see this, consider now the term $(k - k^W) = (1 - \sigma)(k - k^*)$ in equations (19) and (20), where $\sigma = L/L^W$ is Home's share of global labor endowment. Notice that under free trade (and factor–price equalization) the difference in factor abundance between Home and Foreign equals the corresponding difference in per capita incomes:

$$\frac{(1-\sigma)}{r}[(k-k^*)]=\frac{(1-\sigma)}{r}[(w+rk)-(w+rk^*)]=\frac{(1-\sigma)}{r}(I-I^*).$$

Substituting this expression into equations (19) and (20) yields

$$F_K - F_K^H = w(1-\sigma)\frac{a_{L0}LL^W}{(rK^W+wL^W)}\left(\frac{K^W}{L^W}-\frac{a_{K0}}{a_{L0}}\right)(I-I^*), \tag{21}$$

$$F_L - F_L^H = -r(1-\sigma)\frac{a_{L0}LL^W}{(rK^W+wL^W)}\left(\frac{K^W}{L^W}-\frac{a_{K0}}{a_{L0}}\right)(I-I^*). \tag{22}$$

Equations (21) and (22) deliver a per capita income version of Theorem 3. For instance, if clothing (the luxury good) is capital intensive, the capital-abundant rich country (Home) exports capital services. In this case the factor content of trade under homothetic preferences exceeds the factor content of trade under quasilinear preferences. The opposite result holds if clothing is labor intensive. Notice also that this difference (in the factor content of trade) is proportional to the *per capita income* difference between the two countries and to the difference in factor intensities between the two goods.

The per capita income version of Theorem 3 offers a possible explanation for the "missing trade" mystery, which is associated with the seminal work of Trefler (1995). He analyzed the empirical relevance of the multicountry, multifactor versions of equations (17) and (18) using a sample of 33 countries and nine factors of production. He calculated the deviations from the HOV theorem for Home as

$$\varepsilon_K = F_K - (K - sK^W) = F_K - F_K^H, \tag{23}$$

$$\varepsilon_L = F_L - (L - sL^W) = F_L - F_L^H, \tag{24}$$

where F_K, F_L are the observed capital and labor components of Home's factor trade and $F_K^H = K - sK^W$, $F_L^H = L - sL^W$ are the predicted components of Home's factor trade under homothetic preferences. Trefler found that factor–service trade (i.e. F_K, F_L) is much smaller than its factor endowment prediction (i.e. $K - sK^W$, $L - sL^W$) and christened this finding "the missing-trade" mystery. He also found that the deviations from the HOV theorem (i.e. ε_K, ε_L) depend on per capita income differences (Trefler, 1995, Fig. 2). His findings have received considerable attention by empirical researchers.[10]

Suppose now that "true" consumer preferences are nonhomothetic and can be modeled by a quasilinear utility function. Assume also that the empirical researcher calculates the predicted factor content of trade under the assumption of homothetic preferences in her attempt to test the traditional HOV proposition. It is obvious then that the calculated deviations from the HOV theorem would be given by equations (21) and (22). And if clothing (the luxury good) were capital intensive, the researcher would confirm the paradox of "missing trade," or, to phrase it more precisely, the assumption of homothetic preferences would amplify the predicted factor content of trade and the magnitude of missing trade. Since, in general, per capita income deviations are larger among poor countries than among rich countries, the magnitude of "missing trade" would be more prevalent among poor countries. This is exactly what Trefler finds in his analysis of the traditional HOV proposition. In his own words, "poor countries tend to have negative deviations and rich countries tend to have positive deviations. The correlation of the number of negative deviations per country with per-capita GDP is 0.87" (Trefler, 1995, p. 1032).

Under the linear-expenditure demand system generated by Stone–Geary preferences, analyzed by Hunter and Markusen (1988) and Chung (2003), among many others, income per capita differences across countries affect the factor content of trade as well and can reverse the pattern of trade in factor services. In the case of quasilinear preferences, equations (15) and (16) exclude this possibility. Therefore, quasilinear preferences offer a "theoretical" correction to the magnitude of missing trade by reducing the factor-content bias of homothetic preferences without affecting the standard comparative advantage theorems. This unique property and the analytical simplicity of quasilinear preferences enhance their value as a tool of general-equilibrium analysis.

5. Concluding Remarks

The present paper took the general-equilibrium properties of quasilinear preferences seriously. At first sight, quasilinear preferences do not look promising as a tool of general-equilibrium analysis: income effects are captured by the luxury good only, and the demand for the other good depends only on its own price. However, we were willing to undertake the task of embedding this preference structure into the standard trade model and discovered several novel properties. Quasilinear preferences generate expenditure shares that depend on per capita income in accordance with the empirical evidence. This property leads, in turn, to a natural partition of goods into luxury and necessities. In addition, surprisingly again, quasilinear preferences preserve all the theorems of comparative advantage which are routinely associated with homothetic preferences.

We also discovered a number of important differences between homothetic and quasilinear preferences. Under perfect competition and incomplete specialization in production, quasilinear preferences generate too little or too much factor-service trade compared to homothetic preferences depending on whether the luxury good is capital or labor intensive. The magnitude of this bias depends on the degree of factor intensity differences between the two sectors and on per capita income differences among countries.

Additionally, we discovered that within a class of nonhomothetic preferences quasilinear preferences exhibit different general-equilibrium properties than Stone–Geary ones. The former cannot reverse the HOV proposition, whereas the latter can. Based on these discoveries, one can reasonably argue that the assumption of quasilinear preferences corresponds to a theoretical "correction" which reduces the magnitude of the "missing-trade" mystery, while preserving the factor proportion theory of comparative advantage!

In summary, one could reasonably argue that quasilinear preferences are more empirically relevant than homothetic preferences because they generate consumption expenditure shares that vary with income per capita in accordance with evidence, and they can reduce the magnitude of missing trade. They are also more desirable than Stone–Geary preferences because they preserve all the theorems of comparative advantage.

Naturally our analysis suggests a few novel extensions. For instance, it is straightforward to introduce Armington-based home bias and to augment the production side of the global economy by adding technological differences across countries. It is also feasible and interesting to analyze the case of many goods and many factors of production and to introduce imperfect competition in product markets. These generalizations constitute very fruitful avenues for future research.

Appendix

Proof of Lemma 1

It suffices to consider the case where $Y_1/L > Y_1^*/L^*$ because the same argument applies to the case where $Y_1/L < Y_1^*/L^*$. The market-clearing condition (4) can be rewritten as:

$$L\left[\frac{Y_1}{L} - D(p)\right] = L^*\left[D(p) - \frac{Y_1^*}{L^*}\right]. \tag{A1}$$

Equation (A1) implies that the signs of the terms in square brackets must be the same.
Consequently, there are two possible cases:

$$\frac{Y_1}{L} - D(p) > 0, \quad \frac{Y_1^*}{L^*} - D(p) < 0 \quad \text{and} \quad \frac{Y_1}{L} - D(p) < 0, \quad \frac{Y_1^*}{L^*} - D(p) > 0. \tag{A2}$$

However, the latter sign pattern is excluded because it contradicts the assumption $Y_1/L > Y_1^*/L^*$. Therefore, the former sign pattern is the only one consistent with our initial assumption and implies that Home has a positive export supply of good 1 and Foreign has a positive import demand for good 1. Therefore, Home exports good 1 and imports good 2 to maintain balanced trade. □

Derivations of Equations (15) and (16)

To derive equations (15) and (16), we need two auxiliary equations. The first is:

$$Lq_1 = LD = \frac{L}{L + L^*}(L + L^*)D = \sigma Y_1^W, \tag{A3}$$

which comes from the definition of σ and the world market-clearing condition of good 1: $(L + L^*)D = Y_1 + Y_1^* \equiv Y_1^W$. The second is the "world" national income equality:

$$rK^W + wL^W = pY_1^W + Y_0^W. \tag{A4}$$

Making use of equations (A3) and (A4), Home's capital content of trade is given by

$$\begin{aligned}
F_K &\equiv a_{K1}(Y_1 - Lq_1) + a_{K0}(Y_0 - Lq_0) \\
&= a_{K1}Y_1 + a_{K0}Y_0 - a_{K1}Lq_1 - a_{K0}Lq_0 \\
&= K - a_{K1}\sigma Y_1^W - a_{K0}(rK + wL - p\sigma Y_1^W) \\
&= K - \sigma(K^W - a_{K0}Y_0^W) - a_{K0}[rK + wL - \sigma(rK^W + wL^W - Y_0^W)] \\
&= K - \sigma K^W - a_{K0}[r(K - \sigma K^W) + w(L - \sigma L^W)] \\
&= (1 - ra_{K0})(K - \sigma K^W) \\
&= wa_{L0}(K - \sigma K^W),
\end{aligned} \tag{A5}$$

which is equation (15). The second equality above follows from full employment of capital in Home, i.e. $a_{K1}Y_1 + a_{K0}Y_0 = K$ and the last equality utilizes the profit-maximization condition in sector 0, such that $ra_{K0} + wa_{L0} = 1$.

Similarly, Home's labor content of trade is derived as follows:

$$\begin{aligned}
F_L &\equiv a_{L1}(Y_1 - Lq_1) + a_{L0}(Y_0 - Lq_0) \\
&= a_{L0}Y_0 + a_{L1}Y_1 - a_{L1}Lq_1 - a_{L0}Lq_0 \\
&= L - a_{L1}Lq_1 - a_{L0}(rK + wL - p\sigma Y_1^W) \\
&= L - \sigma(L^W - a_{L0}Y_0^W) - a_{L0}[rK + wL - \sigma(rK^W + wL^W - Y_0^W)] \\
&= (L - \sigma L^W) + \sigma a_{L0}Y_0^W - a_{L0}[r(K - \sigma K^W) + w(L - \sigma L^W) - \sigma Y_0^W] \\
&= -ra_{L0}(K - \sigma K^W) + (1 - wa_{L0})(L - \sigma L^W) \\
&= -ra_{L0}(K - \sigma K^W),
\end{aligned} \tag{A6}$$

where equations $L - \sigma L^W \equiv 0$, $ra_{K0} + wa_{L0} = 1$, and $GDP = rK + wL = Lq_0 + pLq_1$ were used as well. □

Derivation of Equations (19) and (20)

Subtract equation (17) from (15) to obtain

$$\begin{aligned}
F_K - F_K^H &= wa_{L0}(K - \sigma K^W) - (K - sK^W) \\
&= wa_{L0}\left(K - \frac{L}{L^W}K^W\right) - \left(K - \frac{rK + wL}{rK^W + wL^W}K^W\right) \\
&= w(KL^W - LK^W)\left(\frac{a_{L0}}{L^W} - \frac{ra_{K0} + wa_{L0}}{rK^W + wL^W}\right) \\
&= wLL^W\left(\frac{K}{L} - \frac{K^W}{L^W}\right)\frac{ra_{L0}}{rK^W + wL^W}\left(\frac{K^W}{L^W} - \frac{a_{K0}}{a_{L0}}\right) \\
&= \frac{rwa_{L0}LL^W}{rK^W + wL^W}\left(\frac{K}{L} - \frac{K^W}{L^W}\right)\left(\frac{K^W}{L^W} - \frac{a_{K0}}{a_{L0}}\right),
\end{aligned} \tag{A7}$$

where the second equation uses the definition of σ and s, and the third equation uses the zero-profit condition $ra_{K0} + wa_{L0} = 1$.

Subtract equation (18) from (16) to obtain

$$\begin{aligned}
F_L - F_L^H &= -ra_{L0}(K - \sigma K^W) - (L - sL^W) \\
&= -ra_{L0}\left(K - \frac{L}{L^W}K^W\right) - \left(L - \frac{rK + wL}{rK^W + wL^W}L^W\right) \\
&= -r(KL^W - LK^W)\left(\frac{a_{L0}}{L^W} - \frac{ra_{K0} + wa_{L0}}{rK^W + wL^W}\right) \\
&= -rLL^W\left(\frac{K}{L} - \frac{K^W}{L^W}\right)\frac{ra_{L0}}{rK^W + wL^W}\left(\frac{K^W}{L^W} - \frac{a_{K0}}{a_{L0}}\right) \\
&= -r\frac{ra_{L0}LL^W}{rK^W + wL^W}\left(\frac{K}{L} - \frac{K^W}{L^W}\right)\left(\frac{K^W}{L^W} - \frac{a_{K0}}{a_{L0}}\right),
\end{aligned} \tag{A8}$$

where the second equation uses the definition of σ and s, and the third equation uses the zero-profit condition $ra_{K0} + wa_{L0} = 1$. □

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Notes

1. These determinants include cross-country differences in labor productivity, relative factor abundance, and scale economies. In addition, early international trade theorists assumed identical and homothetic tastes to rule out multiple pre-trade equilibria and other pathologies in the definition of factor abundance (see Jones, 1956; Bhagwati, 1967; Inada, 1967).
2. Deaton and Muellbauer (1980) provide a summary of earlier empirical work that rejects homotheticity. Hunter and Markusen (1988) found highly significant deviations from

homotheticity. Hunter (1991) concluded that nonhomothetic preferences could account for more than 25% of interindustry trade flows.
3. In an earlier paper, Reimer and Hertel (2003) argued that a nonhomothetic model of demand explains about 20% of the "missing trade" mystery. More recently, Reimer and Hertel (2010) concluded that nonhomothetic preferences explain a relatively small portion of the "missing-trade" paradox because the magnitude of missing trade depends not only on consumption patterns but on production techniques as well, both of which vary systematically across countries with per capita income.
4. Johnson (1959) was the first to introduce differences in the propensity to consume between capitalists and workers and examine the effects of this assumption on trade patterns. The nonhomothetic linear–expenditure system, which is based on Stone–Geary preferences, has been utilized to address the role of per capita income as a determinant of trade patterns and trade volumes (Hunter and Markusen, 1988; Hunter, 1991; Chung, 2003, among others). Ruffin (1977) and more recently Mitra and Trindade (2005) introduced nonhomothetic tastes in the standard trade model to analyze the nexus between income inequality within countries and trade flows.
5. In a companion paper, Doi et al. (2004) have addressed similar questions concerning the pattern of trade under quasilinear preferences and under the assumption that each of the two countries has the same size measured by the number of consumers. The present paper complements the analysis of Doi et al. (2004) by considering the case of two countries with arbitrary labor endowments. Unlike Doi et al. (2004), we calculate the factor-content of trade, and establish an exact (as opposed to a modified) version of the HOV theorem.
6. In the case of many goods, a popular representation of quasilinear preferences is captured by the utility function $U \equiv q_0 + \sum_j u(q_j)$, where index j denotes commodities and $u(q_j)$ is an increasing and concave function of q_j. The main results of the present paper can be readily generalized to address the case of many factors and many commodities.
7. Loosely speaking, one can think of clothing as the sector with the high-income demand elasticity and of food as the sector with the low-income demand elasticity, where the term "low" takes the extreme value of zero. For instance, Reimer and Hertel (2003, Table 1) report expenditure elasticities that vary from 0.403 for the category of grains and other crops to 1.337 for the category of financial and business services. This evidence suggests that the effects of income changes on demand are highly asymmetric across sectors.
8. For instance, Markusen (1986, p. 1003) reports that, in 1982, expenditures on food ranged from 60% of personal consumption expenditure in India to 13% in the United States. He also reports that the share of food expenditure follows a declining convex graph as a function of per capita income in a sample of 34 countries (see his Fig. 1 on p. 1004). Hunter (1991, Table 1) confirms Markusen's findings and reports that the share of income spent on consumption of medical care, transportation and communication, fuel and power, and education increases in per capita income.
9. Using the notation of the present paper, Stone–Geary preferences are represented by the utility function $U = (q_0 - \bar{q}_0)^{\alpha}(q_1 - \bar{q}_1)^{1-\alpha}$, where $\bar{q}_0$ and $\bar{q}_1$ are positive parameters interpreted as minimum consumption requirements and $0 < \alpha < 1$ is also a parameter. The demands for goods 0 and 1 are given by $q_0 = \bar{q}_0 + \alpha(I - p_0\bar{q}_0 - p_1\bar{q}_1)/p_0$ and $q_1 = \bar{q}_1 + (1-\alpha)(I - p_0\bar{q}_0 - p_1\bar{q}_1)/p_1$ where I is per capita income. It is obvious from the functional forms of these demand curves that even if the two countries have identical per capita production of both goods, income per capita differences can generate different autarky prices and lead to trade.
10. See, for instance, the seminal contributions on the empirics of the factor content of trade by Trefler (1993, 1995) and Davis and Weinstein (2001, 2003) for an overview of the literature.