THE CONUNDRUM OF RECOVERY POLICIES: GROWTH OR JOBS?

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Abstract: This paper adopts a Neo-Schumpeterian approach to analyze the relationship between unemployment and growth. We propose a model that features creative destruction, fully-endogenous growth, and search-based unemployment. We find that adverse shocks to profit flows and reduced matching efficiency can lead to low growth and high unemployment. We analyze the effects of several recovery policies used by the governments. Industrial policies in the form of production subsidies to young small firms, production taxes to old large firms, and R&D subsidies to entrepreneurs all stimulate growth but also increase unemployment. In contrast, recovery policies reducing job vacancy costs lead to higher growth and lower unemployment.

Keywords: Economic growth, equilibrium unemployment, recovery policies, vacancy creation.

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1 Introduction

The Great Recession has generated high persistent unemployment among several advanced countries (OECD, 2011), and thus has sparked the debate of how governments create jobs (Blinder, 2009); and, more generally, how they identify and implement effective recovery policies. Caballero (2010, p. 96) states that crises appear to be inevitable and unpredictable. As a result, he suggests macroeconomists emphasize crafting appropriate recovery policy responses.

In this paper, we construct a fully-endogenous model of growth and unemployment and investigate the effects of recovery policies. Which policies unequivocally stimulate growth and reduce unemployment? Which policies generate tradeoffs between growth and jobs, presenting a conundrum for the policy maker? Are there any policy combinations that lead to lower unemployment and higher growth?

Our setting is a dynamic, general-equilibrium model of Schumpeterian growth, and equilibrium unemployment with the following main features.[1] First, growth is endogenously driven by deliberate innovation efforts of entrepreneurial firms. Innovators discover production techniques that lower costs by a fixed percentage. The arrival of innovations is governed by a stochastic Poisson process. It generates fully-endogenous growth of total factor productivity (TFP) and per-capita output. An innovator enjoys temporary monopoly profits fueling investments in R&D.

Second, innovators face labor market frictions. They must engage in a stochastic search process to find, organize, and train workers prior to starting production at full capacity. The matching process requires the creation, maintenance, and management of costly job vacancies. Firms optimize the amount of vacancies based on profit maximization. Matching takes place between blocks of vacant positions and workers. As in the case of innovations, the arrival of job-matches is also governed by a stochastic Poisson process. The endogenous arrival of new technologies together with labor market frictions gives rise to search-based unemployment of the Diamond-Mortensen-Pissarides (DMP) type.[2]

Third, firms undertake Rent-Protection Activities (RPAs) to discourage innovation efforts of potential competitors, with a view to prolonging monopoly tenure and delaying the emergence of new technology leaders. We assume that innovation depends directly on R&D investment and inversely on RPAs. In this

[1] The term “Schumpeterian growth” refers to endogenous growth generated through the process of creative destruction, as described by Schumpeter (1934).

[2] Costly vacancy creation and stochastic block (as opposed to individual) matching between firms and job-applicants are two central features that differentiate our work from related Schumpeterian models of growth and unemployment, such as those of Aghion and Howitt (1994), Mortensen (2005), and Şener (2000, 2001).

model, RPAs have two key features. First, RPAs remove the counterfactual scale-effects property from the model resulting in fully-endogenous growth.[3] Second, RPAs are financed by retained earnings and respond to policy changes. Thus, policies that reduce unemployment may hamper growth by increasing the returns to RPAs vis-à-vis R&D. Similarly, policies that increase unemployment may foster growth by raising the relative R&D-RPA returns. As a result, RPAs can be a driving force behind the recovery conundrum: jobless growth or stagnant growth with job-creation.

The central role of RPAs in the conundrum of recovery policies requires a few remarks regarding their nature and empirical relevance. RPAs capture the notion that incumbents adopt resource-using strategies to preserve their economic rents and retard the pace of innovation activities by challengers. Such strategies include investments in trade secrecy; increasing technological complexity of their products; expenditures on lobbying politicians to provide stronger protection of intellectual property; and expenditures on patent-infringement litigation. For instance, Coca Cola spends substantial resources to keep its formula secret; Intel has been producing smaller and more complex microprocessors that are more difficult to reverse engineer. According to Chu (2008), the U.S. pharmaceutical industry spent more than one billion dollars on lobbying the government during the period 1998-2006. Chu (2008, Table 2) documents the various laws enacted by the US Congress that have led to an extension of the commercial lifetime of pharmaceutical patents. In addition, Bessen and Meurer (2012) estimate the private costs of patent litigation using stock market event studies. During 1984-99 alleged patent infringers incurred expected costs of over $16 billion per year. The ratio of annual litigation costs to aggregate R&D expenditure averaged to 14% during 1996-1999. In summary, RPAs take a variety of forms and account for a substantial fraction compared to R&D expenditures.[4] Following the modeling approach of Dinopoulos and Syropoulos (2007), we abstract from possible differences in the nature of RPAs. We model RPAs as a single activity that is undertaken by incumbent firms only and aim at reducing the productivity of R&D investments by potential competitors. Accordingly, R&D may become more difficult because incumbent firms may allocate more resources to RPAs.

The model's steady-state equilibrium is unique, and entails the simultaneous presence of search-based unemployment as well as Schumpeterian growth. The expected life of a firm is finite, and consists of four

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[3] The removal of scale effects further distinguishes our paper from Aghion and Howitt (1994) and Mortensen (2005). Ha and Howitt (2007), Madsen (2007, 2008), Ang and Madsen (2011), Venturini (2012a) among others, argue that fully-endogenous growth theory is more empirically relevant than semi-endogenous growth theory. For arguments in favor of semi-endogenous growth theory, see Jones (2005) and Venturini (2012b).

[4] Dinopoulos and Syropoulos (2007), Şener (2008), and Grieben and Şener (2009) among others provide additional evidence and theoretical applications on RPAs.

distinct, consecutive stages. The length of each stage is stochastic and endogenous. In the R&D phase, firm size is indeterminate, i.e., each firm is infinitesimally small. Upon discovering a new process innovation, a firm becomes a young technology leader, captures an exogenous and small share of the market, and enters the vacancy-creation process. It advertises new positions, interviews prospective workers, develops distribution systems, trains and organizes workers and suppliers. This matching process is stochastic and upon completion, the firm expands production and enters adult stage. The adult firm immediately captures the whole market. It is then targeted by outside innovators, and engages in RPAs to delay the emergence of a new technology leader. Innovation success by an outside firm implies that the adult firm enters its old stage, during which it becomes a technology follower competing against a young technology leader. As an old firm, it still captures a large part of the market. It does not however engage in RPAs and will eventually be replaced by a new technology leader upon successful matching by that firm.

The model generates two types of industries, referred to as A and B industries. Type A industries consist of adult firms that serve the whole market and engage in RPAs. They are targeted by prospective innovators. Type B industries consist of young and old firms. In a B industry a young technology leader tries to replace an old technology follower by creating more jobs through costly vacancies and stochastic matching. In other words, small, young firms create jobs in our model, whereas large, old firms destroy jobs.[5]

We analyze labor markets in the unemployment rate - vacancy rate space following the convention introduced by the DMP literature. In particular, we identify a downward sloping Beveridge Curve by equating labor flows in and out of unemployment. We also establish an upward sloping Vacancy Creation curve by considering the optimal job-creation decisions of firms, which embed optimal R&D and RPA decisions.

What types of shocks can generate higher unemployment and lower growth in our model? We find that adverse shocks to profit flows can lead to lower steady-state growth and higher structural unemployment. In our model, this is captured by a decline in the exogenous size of innovations, which reduces profit margins. The intuition is that a lower profit flow reduces the incentives for job vacancy creation. Hence, the job-finding rate declines and the unemployment rate increases. The decline in vacancy-unemployment ratio implies a higher matching rate for young firms and thus a higher replacement rate for old firms. This

[5] This modeling feature is consistent with Haltiwanger et al. (2013). They argue that firm age is more important than firm size in the process of job creation and destruction.

increases the incentives of adult firms to undertake RPAs, leading to a fall in the relative profitability of R&D vis-à-vis RPAs. In the new equilibrium, the rates of innovation and growth decline.

What type of recovery policies can be used to lower unemployment and raise growth? In this respect, we depart from traditional macroeconomic instruments by first considering *targeted industrial policies*. Production subsidies for adult firms reduce unemployment but also lower growth, implying a trade-off between jobs and growth. Production subsidies for young firms and R&D subsidies for entrepreneurs increase growth but also raise unemployment, implying again the same trade-off. We thus conclude that such targeted industrial policies present a *conundrum* in the context of recovery since they imply a positive relationship between growth and unemployment. Simulation analysis suggests that certain policy combinations can actually eliminate the trade off. For example, a combination of production subsidies targeting adult and young firms, or a combination of R&D subsidies for entrepreneurs and production subsidies targeting adult firms can stimulate growth and reduce unemployment. In contrast to the targeted industrial policies, we find that *a broad-based labor market policy* that facilitates job-matching by reducing the cost of vacancy creation can reduce unemployment and increase growth.

What is the intuition behind these results? The dynamic nature of our model implies that the differentials between market valuations of R&D, young, adult and old firms determine the incentives for each activity, R&D, RPA and job-creation. The intensities of these activities determine ultimately the equilibrium levels of growth and unemployment. Consider for example subsidizing adult firms. These firms are fully-matched monopolies and do not actively engage in job-creation. So, why do increased subsidies to adult firms lead to more job creation and lower unemployment? The reason is that young firms, as agents of job creation, look up to the value of adult firms when they make their job-openings decisions. Production subsidies given to adult firms raise the valuations of adult firms. This strengthens the incentives of young firms to become adult firms via vacancy creation and matching. This is the major force driving down the unemployment. At the same time, the rise in market valuations of adult firms strengthens their incentives to engage in rent protection, simply because they have more at stake to lose whenever further innovation arrives. The resulting increase in the relative profitability of RPAs vis-à-vis innovation leads to lower equilibrium rates of innovation and growth. Such “trickle-down incentive” effects exclusively stem from dynamic modeling approach that models the life-cycle of firms. In Section 4, we provide a detailed discussion of how each policy change affects incentives and equilibrium rates of unemployment and growth.[6]

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[6] Our labeling of this mechanism as “trickle down of incentives” is in the spirit of Acemoglu and Akcigit (2012)

Our focus on recovery policies complements several recent policy-oriented research papers that investigate the Great Recession through the lens of the standard DMP models.[7] Daly et al. (2012), in particular, emphasize the importance of a general-equilibrium framework to understand unemployment changes; and stress that much of the public discussion has focused on the Beveridge Curve with little emphasis on the Job Creation Curve. Similar to our paper, these papers also consider adverse shocks that reduce profit flows and thus job-creation incentives as the source of recession. However, in these settings, the mechanics of growth are not incorporated and the job-destruction rate is usually exogenous by assumption. In our model policies affect employment levels by simultaneously impacting the rate of job destruction and vacancy creation.

The remainder of the paper is organized as follows. Section 2 describes the elements of the model. Section 3 derives the equilibrium conditions formally and illustrates the equilibrium graphically. Section 4 analyzes the growth and employment effects of several policies. Section 5 briefly comments on welfare effects of recovery policies. And Section 6 offers concluding remarks. Algebraic derivations are relegated to various appendices.

2 The Model

Our model borrows its elements of growth and rent protection from the full-employment model of Dinopoulos and Syropoulos (2007). We incorporate labor-market frictions following the DMP literature (Pissarides, 2000). There exist two main differences between the approach of the present paper to modeling unemployment and the standard DMP literature. First, while the DMP literature relies on the neoclassical growth model and exogenous idiosyncratic shocks to generate labor turnover and unemployment, our model employs an endogenous job-destruction mechanism linked to endogenous technological change. Second, our modeling of search and matching frictions is related to a recent line of research which combines costly vacancy creation and block-matching, where an endogenous measure of workers is matched with a firm instead of one-to-one matching between firms and workers. Seminal studies include Blanchard and Gali (2010), Helpman and Itskhoki (2010), and Helpman et al. (2010). In our model, the combination of costly vacancy creation and stochastic block-matching renders unnecessary any bargaining between a worker and a firm. As a result, we are able to maintain the assumption of

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who analyze the effects of intellectual property rights policy in a setting where both leader firms and followers perform R&D. They find that subsidizing leader firm's research could be more effective than subsidizing followers, because followers look up to the valuation obtained by leaders when they do R&D.

[7] See Lubik (2013), Kitao et al. (2011), Daly et al. (2012), Pissarides (2011) and also Mortensen (2011, pp. 1084-1090).

competitive labor markets used routinely in endogenous growth theory.[8]

Our paper is also closely related to the work of Aghion and Howitt (1994) and Mortensen (2005), who endogenize job-destruction in the context of Schumpeterian growth and find an ambiguous relationship between growth and unemployment. However, we differ in a number of aspects. First, we depart from Aghion and Howitt (1994) who establish the ambiguity result in the context of *exogenous* growth. When they consider endogenous growth, they find that changes in the frequency of innovations have *no impact* on the rate of unemployment. In this case, only one parameter, innovation size, affects growth, and they find that whenever growth increases, unemployment also increases. In contrast, our fully-endogenous growth model generates either a positive or a negative long-run correlation between unemployment and growth depending on the policy change or the exogenous shock considered. This finding is consistent with empirical evidence summarized by Postel-Vinay (2002, p. 740).

Second, our model differs from Mortensen's (2005) model in which vacancy creation is assumed to be costless and thus policies do not affect employment through vacancy-creation.[9] Third, in contrast to Aghion and Howitt (1994) and Mortensen (2005), here the matching rate of young firms by itself contributes to the endogenous job-destruction process. This is because we consider a stepwise matching process: an innovator immediately captures a small portion of the market, and then undertakes another step that involves block matching to drive out the incumbent firm.[10]

2.1 Consumers

The economy consists of a continuum of identical and infinitely-lived households whose measure is set equal to one. The size of each household is denoted by N and remains constant over time. Given the unit measure of households, the size of aggregate population also equals N.[11] Each household member inelastically supplies one unit of labor per period of time. The representative household maximizes the infinite horizon utility

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[8] With block matching, the individual applicant has no bargaining power. We consider this as a realistic feature of our model because most unemployed workers are not organized in labor unions, and coordination among job applicants does not occur in practice.

[9] In Mortensen (2005), the bargaining solution between each firm and each worker substitutes the firm's choice of profit-maximizing vacancies.

[10] We also differ from Mortensen and Pissarides (1994), who propose a model with endogenous job-destruction and exogenous productivity shocks. The latter serve as a source of job destruction, whereas in the present model endogenous technological change destroys jobs.

[11] Allowing for positive population growth leaves most key results intact. The assumption of a constant level of population is more consistent with a medium or short-run interpretation of the steady-state equilibrium.

$$H = \int_0^\infty e^{-\rho t} \log h(t) dt , \qquad (1)$$

where $\rho > 0$ is the subjective discount rate. The subutility function *log h*(*t*) is defined as

$$\log h(t) \equiv \int_0^1 \log y(\omega, t) d\omega , \qquad (2)$$

where $y(\omega, t)$ is the per-capita demand for goods manufactured in industry ω at time *t*. The economy consists of a continuum of structurally-identical industries indexed by $\omega \in [0, 1]$. Household optimization can be viewed as a two-stage problem. In the first stage each household allocates consumption expenditure to maximize *h*(*t*) for any given product prices. Since goods enter the subutility function symmetrically, each household spreads its per-capita consumption expenditure *c*(*t*) evenly across all available goods. Thus, demand for each good equals

$$Y(\omega, t) = c(t) N / P(\omega, t) , \qquad (3)$$

where $Y(\omega,t) = y(\omega,t)N$, and $P(\omega,t)$ is the market price of the purchased goods in industry ω at time *t*. From now on, for notational simplicity, we drop the time index *t* where appropriate.

The second stage involves a dynamic optimization problem in which each household chooses the evolution of *c* over time. Substituting (2) into (1) and using *Y* from (3), one can simplify the household's dynamic problem to maximizing $\int_0^\infty e^{-\rho t} \log c \; dt$ subject to the budget constraint $\dot{A} = W + rA - cN$, where *A* denotes the asset holdings of each household, and *W* is household's expected wage income. Variable *r* is the rate of return obtained from a completely diversified asset portfolio. The solution of the dynamic optimization problem provides the Keynes-Ramsey rule:[12]

$$\dot{c}/c = r - \rho . \qquad (4)$$

Because the labor supply and the wage rate are constant in the steady state, equation (4) implies a constant per-capita consumption expenditure measured in units of labor[13] and $r = \rho$ in equilibrium.

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[12] Each household consists of a large number of members who engage in income transfers such that each member enjoys the same level of consumption regardless of individual earnings. This implies the absence of effective uncertainty in individuals' income and consumption emanating from idiosyncratic firm-level risk. Bayer and Wälde (2011) offer a modified version of (4) which takes into account individual income uncertainty.

[13] Nevertheless, the aggregate price index P_{AGG} declines over time whenever innovation takes place as will be shown later. As a result, real per-capita consumption measured in units of final output grows over time.

2.2 Job Creation and Destruction

Labor is the only factor of production. The labor force consists of low-skilled and high-skilled workers. The proportion of the former is given as $1 - s$ and that of the latter is given as $s \in (0, 1)$. Low-skilled workers can be employed in manufacturing only, whereas high-skilled workers can be employed in either R&D or RPAs.[14] We assume that high-skilled workers can find employment instantly without going through a job-matching process. Hence only low-skilled workers are subject to turnover and face the prospect of unemployment.[15]

Consider next the hiring process of an innovator. In each industry, production technology improves through the stochastic arrival of process innovations. We assume that a young technology leader (an entrant) can immediately hire a small number of unskilled workers without engaging in costly search. As a result, it captures an exogenous fraction $\phi \in (0, 1)$ of the market and forces the incumbent to lay off a corresponding number of workers.[16] To capture the remaining fraction $1-\phi$ of the market, an entrant must expand capacity and therefore engage in costly search by posting vacant positions. While the entrant is searching, the incumbent continues to supply a fraction $1-\phi$ of the market.[17] When the entrant completes the hiring process, which occurs with endogenous instantaneous probability q, the incumbent exits the market and all of its remaining workers join the unemployment pool. Further innovation in the industry triggers again the above job creation and job-destruction cycle. Hence, at any point in time,

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[14] By allowing resource mobility between R&D and RPAs, we endogenize the intensity of R&D activity and capture an essential feature of endogenous growth theory. This labor assignment is similar to Dinopoulos and Syropoulos (2007) and also Grieben and Şener (2009). In these papers labor mobility between R&D and manufacturing is assumed, while the portion of labor devoted to RPAs is kept fixed. In contrast, here the portion of labor allocated to manufacturing is fixed but workers face unemployment in this sector.

[15] This is a commonly used assumption in the literature. See, among others, the dynamic growth settings of Şener (2001, 2006) and the static model of Davis (1998). This assumption captures in a simple way the well-established unemployment differential between high-skilled and low-skilled workers (see e.g. Nickell and Bell, 1995 and 1996, for descriptive evidence on seven major OECD countries). Moreover, because vacancy creation is costly, the assumption of costly high-skilled labor matching would create a conflict with the assumption of free entry in R&D activities.

[16] One can view this feature as follows. The technology leader instantaneously attracts a share ϕ of workers from the existing incumbent monopolist. For these workers, switching to the technology leader makes sense because they escape the impending unemployment risk. Alternatively, this feature can be justified by considering a geographically fragmented labor market. Successful innovators knowledgeable about the local market can instantly hire a core group of workers to initiate production but then they have to engage in costly search to expand their production.

[17] This step-wise replacement mechanism follows the spirit of Dinopoulos and Waldo (2005, pp. 141-142) where a successful product innovator instantaneously captures a small share of the market followed by a gradual switch of consumers from the previous-generation product to the state-of-the-art quality product.

young firms create and maintain job vacancies and unemployed workers search for and fill the available job vacancies.

2.3 Industry Structure

The assumptions that all industries are structurally identical and that only adult firms are targeted by challengers engaged in R&D imply that, at each point in time, there are two possible industry configurations which we refer to as A and B industries. In A industries, there is an adult technology leader serving the entire market and entrepreneur firms that invest in R&D to discover the next process innovation. At the same time, each adult firm engages in RPAs to protect its monopoly profits by deterring the innovation effort of challengers. The reader can think of industries A as “growth-oriented” industries because they are targeted by future innovators. In B industries, there is a technology follower in its old phase serving a fraction $1-\phi$ of the market, and a young technology leader with the state-of-the-art production process, serving a fraction ϕ of the market and thereby exerting partial monopoly power. At the same time, the new technology leader is searching to hire workers and drive the old technology follower completely out of business. One can think of B industries as “employment-oriented” industries: each young firm invests in vacancy maintenance and hiring of new workers aiming at expanding capacity and employment.

Let n_A and $n_B = 1 - n_A$ represent the fraction (measure) of A and B industries, respectively. Let also $I(\omega) = I$ denote the intensity of the Poisson process that governs the arrival of innovations in each industry. An A industry switches to a B industry with instantaneous probability Idt. Hence, the expected flow of industries from A into B is $n_A Idt$. When a young firm successfully completes its hiring process, a B industry switches to an A industry. The probability of this event is qdt and hence the expected flow of industries from B into A is $(1 - n_A)qdt$. Consequently, the net flow into the A industries is $dn_A = (1-n_A)qdt - n_A Idt$, which implies

$$\dot{n}_A = q(1-n_A) - In_A . \tag{5}$$

2.4 Product Markets

Manufacturing of final consumer goods uses only low-skilled labor according to a constant returns to scale production function $Y_i = \lambda^{m_i} Z_i$, where Y_i is the output of firm i, $\lambda > 1$ is a parameter capturing the size of each process innovation, integer m_i is the number of process innovations which have occurred until the time of production, and Z_i is the number of low-skilled workers employed. In other words, the term

λ^{m_i} captures the total factor productivity (TFP) component of production.

Let $\lambda^{m(\omega)}$ represent the state-of-the-art productivity level in industry ω. Consider an adult firm in an A industry that has access to the state-of-the-art m^{th} technology and has completed the hiring process. For this firm, the marginal (and average) cost of manufacturing one unit of final goods is $w_L / \lambda^{m(\omega)}$, where w_L is the wage rate of low-skilled labor. Hence $1/\lambda^{m(\omega)}$ measures the amount of low-skilled labor required per unit of output in industry ω.

The adult firm competes against a follower with access to technology one step down the technology ladder, i.e. the $[m(\omega) - 1]^{\text{th}}$ technology, and a unit cost of $w_L / \lambda^{m(\omega)-1}$. These firms compete in a Bertrand fashion: the technology leader uses its cost advantage to engage in limit pricing and capture the entire market. In equilibrium, the adult firm in an A industry charges a price $P_a(\omega) = w_L / \lambda^{m(\omega)-1}$ and incurs a unit cost $w_L(1-\sigma_a)/\lambda^{m(\omega)}$, where $0 < \sigma_a < 1$ ($\sigma_a < 0$) is the adult firm's production subsidy (tax) rate.[18] The adult firm captures the entire market demand $cN/P_a(\omega)$. Thus, in an A industry, an adult firm earns a flow of monopoly profits

$$\pi_a = \frac{cN}{w_L/\lambda^{m-1}}\left[\frac{w_L}{\lambda^{m-1}} - \frac{w_L}{\lambda^m}(1-\sigma_a)\right] = \frac{cN\left[\lambda - (1-\sigma_a)\right]}{\lambda}. \tag{6}$$

The demand for low-skilled labor engaged in manufacturing equals[19]

$$Z \equiv \frac{cN}{\lambda w_L}. \tag{7}$$

Hence, the incumbent's profit flow and labor demand are independent of m, the number of cumulative innovations used for production at time t, but depend on λ, the size of process innovations.

While an adult technology leader earns monopoly profits, it simultaneously invests in RPAs employing high-skilled labor at a wage rate of w_H. The cost of producing X units of RPAs is $w_H\gamma X$, where

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[18] Specifically, the low-cost adult firm can charge price $P_a(\omega) = \left[w_L/\lambda^{m(\omega)-1}\right] - \varepsilon$, where $\varepsilon \to 0$. The high-cost firm can charge a price as low as its marginal cost $w_L/\lambda^{m(\omega)-1}$; however, this price does not generate positive demand and forces the high-cost firm to exit the market. We assume that followers (previous technology leaders) retain the capacity to produce using their own technology and rehiring their old workers without going through costly worker search again. Thus, they face zero capacity maintenance costs, impose a constant threat to enter the market, and force low-cost producers to engage in limit pricing.

[19] Labor demand is given by output produced $cN/P_a(\omega)$ times the unit-labor requirement $1/\lambda^{m(\omega)}$.

γ is the unit-labor requirement of such activities. Hence, the profit flow net of rent protection costs earned by an adult firm is given by

$$\pi_a^{net} = \pi_a - w_H \gamma X \ . \tag{8}$$

Consider now a typical B industry where there are two producing firms: a low-cost young firm with state-of-the-art $m(\omega)^{th}$ technology that serves a portion ϕ of the market; and a high-cost old firm with $[m(\omega) - 1]^{th}$ technology supplying the remaining portion $1 - \phi$ of the market. The profit flow of a young firm is equal to $\phi\pi_y$. In order to determine π_y, note that in B industries, a young firm having access to $m(\omega)^{th}$ technology competes in a Bertrand fashion with the old firm having access to the $[m(\omega) - 1]^{th}$ technology. In equilibrium, a young firm charges limit price $P_y(\omega) = w_L / \lambda^{m(\omega)-1}$, faces market demand $\phi cN/P_y(\omega)$, and incurs unit-cost $\frac{w_L}{\lambda^{m(\omega)}}\left(1-\sigma_y\right)$, where $0 < \sigma_y < 1$ ($\sigma_y < 0$) is the young firm's production subsidy (tax) rate. The follower exits in the ϕ portion of the market. The typical young firm does not invest in RPAs since its technology is not (yet) targeted by entrepreneurs. Thus, in a B industry, the profit flow earned by a young firm is given by

$$\phi\pi_y = \phi \frac{cN}{w_L / \lambda^m}\left[\frac{w_L}{\lambda^m} - \frac{w_L}{\lambda^{m+1}}(1-\sigma_y)\right] = \frac{\phi cN\left[\lambda - (1-\sigma_y)\right]}{\lambda}. \tag{9}$$

Note that $\pi_a = \pi_y$ for $\sigma_y = \sigma_a$.

In B industries, each old firm with the $[m(\omega) - 1]^{th}$ technology can still retain its profit flow in a portion $1 - \phi$ of the market due to labor market frictions. In this segment of the market, an old firm competes in a Bertrand fashion against another firm with access to the $[m(\omega) - 2]^{th}$ technology. An old firm in a B industry charges a price equal to the marginal cost of the rival firm $P_o(\omega) = w_L / \lambda^{m(\omega)-2}$ and incurs a unit cost $w_L / \lambda^{m(\omega)-1}$. We assume that the subsidy or tax rate σ_a no longer applies to old firms. Thus, an old firm in a B industry earns the profit flow $\pi_o = (1 - \phi)cN(\lambda - 1)/\lambda$.

2.5 Job Vacancies and Matching

In B industries, young technology leaders hold vacancies in order to attract workers. Let V_S represent the market valuation of a successfully-matched vacancy, i.e., the expected discounted value of profits per worker employed. Let V_i denote all vacancies created by a young firm i. Let us also denote with α the flow cost of holding a vacancy, which can be interpreted as a fixed recruitment cost that the firm incurs regardless of whether a job is filled.[20] Let q denote the probability that all vacant positions of a firm are matched. In other words, q is the probability that a young firm in a B industry becomes an adult firm serving an entire A industry. Young firm i chooses vacancies V_i to maximize $qV_SV_i - \alpha V_i$. The first term is the expected return from posting V_i vacancies and the second term is the cost of holding those vacancies. The firm takes the matching rate q and the marginal return from vacancy holding V_S as given. Maximizing the above expression with respect to V_i yields the first-order condition $qV_S = \alpha$.

How is V_S determined? Successful matching implies a change in the valuation of a young firm that is given by $V_a - V_y > 0$, where V_a and V_y represent the valuation of an adult firm and a young firm, respectively. Dividing $V_a - V_y$ by the amount of jobs held by an adult firm J_a yields $V_S = (V_a - V_y)/J_a$. In equilibrium, the amount of available jobs (demand for labor) in a given industry must equal the amount of vacancies held by a young firm, that is, $J_a = V_i = (1 - \phi)Z$. All vacancies are subject to the same matching rate (i.e., there is block matching). A young firm does not find it profitable to maintain more vacancies than the number of workers it will employ as an adult firm.[21] Substituting V_S and J_a into the first order condition $qV_S = \alpha$ yields the vacancy creation (VC) condition

$$q\frac{V_a - V_y}{(1-\phi)Z} = \alpha \qquad \textbf{VC}, \qquad (10)$$

where the LHS is the firm-specific expected benefit from holding a vacancy, and the RHS is the cost of maintaining a vacancy.[22]

Next, we establish a link between the firm-specific vacancy matching rate q and aggregate labor market conditions. Let $V \equiv \sum V_i$ represent the level of economy-wide vacancies and U the level of economy-wide unemployment. The arrival of successful job matches is governed by a stochastic process whose intensity is given by the matching function $M(U, V)$. We assume that the matching function is

[20] Pissarides (1985) interprets vacancy costs as (fixed) opportunity costs of machines (capital) required for new job openings. We model vacancy maintenance costs as fixed costs following the standard search unemployment literature.

[21] If a young firm opens more vacancies than the number of workers employed by an adult firm, then the return to holding an extra vacancy drops down to zero. Specifically, $V_S > 0$ for $V_i \in [0, (1 - \phi)Z]$, whereas $V_S = 0$ for $V_i > (1 - \phi)Z$. Note also that although the first-order condition $qV_S = \alpha$ leaves firm-level vacancies indeterminate, it must hold for a finite level of vacancies.

[22] Equation (10) is a knife-edge equilibrium condition which implies an adjustment process linked to changes in matching rate q. Consider, for example, an increase in the marginal return of vacancy creation $(V_a - V_y)/[(1 - \phi)Z]$. This encourages young firms to offer more vacancies. For any aggregate unemployment rate, the excess supply of vacancies makes it more difficult for young firms to attract workers. Thus the firm-specific matching rate q declines to restore equilibrium.

concave, homogeneous of degree one and increasing in both arguments in accordance to the DMP literature.[23]

Let $\theta \equiv V/U$ denote the number of vacancies per unemployed worker capturing labor-market tightness. Dividing $M(U, V)$ by V yields the matching (hiring) rate of young firms $q(\theta) = M(U/V, 1) = M(1/\theta, 1)$. Similarly, dividing $M(U, V)$ by U yields the job-finding rate of unemployed workers $p(\theta) = M(1, V/U) = M(1,\theta)$. Note that $q(\theta)$ and $p(\theta)$ are stochastic Poisson arrival rates, unlike the deterministic rates in Aghion and Howitt (1994).[24] Observe that $\partial q(\theta)/\partial\theta < 0$, that is, as vacancies per unemployed worker increase, it becomes more difficult for firms to fill their vacant positions. Observe also that $\partial p(\theta)/\partial\theta > 0$, that is, as vacancies per unemployed worker increase, unemployed workers can find jobs more easily. The transition rates $p(\theta)$ and $q(\theta)$ satisfy $p(\theta)U = q(\theta)V = M(U, V)$. Constant returns to scale matching technology and symmetry among all young firms imply that at equilibrium $p(\theta)U_i = q(\theta)V_i = M(V_i, U_i)$, where $\theta = V/U = V_i/U_i$ with U_i and V_i denoting firm-specific job-applicants and job openings, respectively. Since the economy-wide and firm-specific labor market tightness levels are the same, $V/U = V_i/U_i$, firm-specific vacant positions are matched at the rate of $q(\theta)$.

In words, young firms engage in limit-pricing and block-matching to fill their vacant positions. Limit-pricing determines the number of vacancies that are consistent with the demand for labor in each industry. Each young firm maintains the targeted measure of vacancies until they are filled and faces uncertainty as to the duration of the hiring process. Accordingly capacity expansion is lumpy, reflecting underlying non-convexities in the production process, and is governed by a Poisson stochastic process with intensity equal to the matching rate $q(\theta)$. We assume that measure of workers hired is infinitesimally small and workers do not take into account how their hiring affects the firm's marginal revenue. This assumption implies that bargaining between workers and firms is not necessary in contrast to the case of matching between one firm and one worker.[25] Accordingly, the absence of bargaining simplifies the analysis and implies that the market for low-skilled workers is competitive. Low-skilled employed workers receive the

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[23] The matching function has the following additional properties: $M(0, V) = M(U, 0) = 0$, $\lim_{x\to 0} \partial M/\partial x = +\infty$, and $\lim_{x\to\infty} \partial M/\partial x = 0$, $x \in \{U, V\}$.

[24] See Pissarides (2000, chapter 6), Aghion and Howitt (1998, section 4.5) and Helpman et al. (2010) for alternative stochastic matching models, where also only a fraction of contacts between workers and open vacancies lead to successful matches.

[25] Helpman et al. (2010) propose a block-matching scheme where each worker is aware of how the firm's revenue depends on hiring that particular worker. Our framework could be extended to incorporate intra-firm bargaining along similar lines.

market wage w_L and unemployed workers receive nothing. General equilibrium interactions determine the equilibrium values of w_L and $p(\theta)$.

2.6 Innovation

Entrepreneurial firms engage in sequential and stochastic R&D races targeting A industries to discover next-generation process innovations and replace adult incumbent firms. The latter engage in RPAs in order to deter innovation efforts of challengers. The intensity of the Poisson process that governs the arrival of innovations for firm *j* is given by

$$I_j = R_j / D \quad \text{with} \quad D = \delta X \ , \tag{11}$$

where R_j represents R&D services of R&D lab *j*, and *D* measures the difficulty of conducting R&D. We model R&D difficulty *D* as a flow variable, where *X* is the level of RPAs undertaken by an incumbent adult firm, and parameter δ is the efficiency of RPAs.

We assume that Poisson arrival rates are independently distributed across firms, industries, and time. Therefore, the industry-wide Poisson arrival rate equals

$$I = \sum_j I_j = \frac{R}{D} \quad \text{with} \quad R = \sum_j R_j \ . \tag{12}$$

2.7 Financial Market

There exists a stock market that channels household savings to firms engaged in R&D. Retained earnings finance RPAs and vacancy maintenance. During a typical R&D race, a firm issues a flow of shares to pay wages of R&D researchers. If a firm wins an R&D race, then it distributes the flow of profits to its stockholders as dividends; if the firm does not win an R&D race, its stockholders receive nothing. The existence of a continuum of industries and the assumption that Poisson arrival rates are independent across firms and over time imply that investors can fully diversify firm-specific idiosyncratic risk by holding an appropriate portfolio. The return to this stock portfolio equals the market interest rate. At each instant in time, there exist distinct stocks issued by R&D labs, young, adult, and old firms. These stocks are traded freely among investors. The absence of profitable arbitrage in the stock market relates the expected equity returns to the effective interest rate of a riskless asset. In what follows we derive the no-arbitrage condition for each of the four stocks.

Let V_R denote the value of a firm engaged in R&D to discover the state-of-the-art process innovation.

The no-arbitrage condition implies that the expected return to any stock issued by an R&D lab must equal the return generated by a fully diversified (idiosyncratic-risk-free) portfolio of equal size. In other words, the expected return of investing V_R in an R&D lab must equal rV_R. The expected income from investing V_R in an R&D lab is calculated as follows. Over a time interval dt, an R&D lab innovates with probability $I_j dt$, becomes a young firm, and realizes a valuation gain $V_y - V_R$. This firm incurs R&D costs $w_H(1-\sigma_R)\beta R_j$, where $0 < \sigma_R < 1$ ($\sigma_R < 0$) is an R&D subsidy (tax) rate, and $\beta > 0$ is the unit-labor requirement of R&D. With probability $(1 - I_j dt)$, however, success does not materialize, and stockholders absorb capital loss $dV_R = \dot{V}_R dt$. Adding these components of equity return, we may write the no-arbitrage condition for an R&D lab as

$$I_j dt\left(V_y - V_R\right) - w_H\left(1-\sigma_R\right)\beta R_j dt + (1 - I_j dt)\dot{V}_R dt = rV_R dt \ . \tag{13}$$

Free-entry in R&D activities drives firm value to zero, i.e., $V_R = \dot{V}_R = 0$. Taking limits in (13) as $dt \to 0$ and using (11) yields the following R&D free-entry condition[26]

$$V_y = \beta\delta w_H\left(1-\sigma_R\right)X \ . \tag{14}$$

Consider now the stock market valuation of a young firm in a B industry. This firm serves a fraction ϕ of the market by employing ϕZ units of labor and realizes profit flow $\phi\pi_y$. At the same time it maintains $V_i = (1-\phi)Z$ vacant positions aiming at expanding its capacity and capturing the entire market. As mentioned earlier, each vacancy costs $\alpha > 0$ to maintain per unit of time. Thus, over time interval dt total costs of holding vacancies are $\alpha(1-\phi)Zdt$. By incurring these costs, a young firm succeeds to complete the hiring process with instantaneous probability $q(\theta)dt$. This firm becomes an adult firm serving the entire market, and moves from industry B to A. Its stockholders realize a capital *gain* $V_a - V_y > 0$. With probability $1 - q(\theta)dt$, no matching occurs. In this case stockholders realize a change in valuation $dV_y = \dot{V}_y dt$. In the absence of stock-market arbitrage opportunities, the expected return generated by investing an amount V_y in stocks issued by a young firm must equal the return of a fully diversified portfolio of equal size rV_y. Collecting terms, we may write the no-arbitrage condition for a young firm as

$$\phi\pi_y dt + q\left(\theta\right)\left(V_a - V_y\right)dt - \alpha\left(1-\phi\right)Zdt + \left[1 - q\left(\theta\right)dt\right]\dot{V}_y dt = rV_y dt \ . \tag{15}$$

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[26] An alternative derivation of R&D free-entry condition is as follows. Consider firm j that is engaged in R&D. During the time interval dt, this firm incurs with certainty a cost of $w_H(1 - \sigma_R)\beta R_j dt$ which corresponds to the subsidized wage bill of employing βR_j researchers. The expected benefit of R&D investment is $V_y I_j dt$. Setting the expected benefit equal to the cost of R&D, and using (11), yields (14).

Taking limits as $dt \to 0$ yields the expression for the stock market value of a young firm

$$V_y = \frac{\phi\pi_y + q(\theta)V_a - \alpha(1-\phi)Z}{q(\theta) + r - \dot{V}_y/V_y}. \tag{16}$$

Next, consider the stock-market valuation of an adult firm. Over a small time interval dt, its stockholders receive dividends equal to the net profit flow $\pi_a^{net} dt = (\pi_a - w_H\gamma X)dt$. With instantaneous probability Idt, further process innovation occurs, and the adult monopolist becomes an old firm in a B industry with valuation V_o. In this case, stockholders of an adult firm absorb capital *loss* $V_a - V_o > 0$. With probability $(1 - Idt)$, no further innovation occurs in the industry. In this case stockholders realize a capital gain $dV_a = \dot{V}_a dt$. Collecting terms, we may write the no-arbitrage condition of an adult firm as

$$(\pi_a - w_H\gamma X)dt - I(V_a - V_o)dt + (1 - Idt)\dot{V}_a dt = rV_a dt. \tag{17}$$

Taking limits as $dt \to 0$ yields the expression for the stock market value of an adult firm

$$V_a = \frac{IV_o + \pi_a - w_H\gamma X}{I + r - \dot{V}_a/V_a}. \tag{18}$$

Finally, consider the stock market valuation of an old firm. This firm is a technology follower and serves $1 - \phi$ fraction of a B-industry market. In a time interval dt, stockholders of an old firm receive $\pi_o dt$ as dividend payments. With probability $q(\theta)dt$, a young technology leader drives an old firm out of the market. In this event, the stockholders of an old firm absorb capital *loss* V_o. With probability $1 - q(\theta)dt$, no matching occurs in the industry. In this event, stockholders realize capital gain $dV_o = \dot{V}_o dt$. Collecting terms, we may write the no-arbitrage condition of an old firm as

$$\pi_o dt - q(\theta)V_o dt + [1 - q(\theta)dt]\dot{V}_o dt = rV_o dt. \tag{19}$$

Taking limits as $dt \to 0$ yields the expression for the stock market value of an old firm

$$V_o = \frac{\pi_o}{q(\theta) + r - \dot{V}_o/V_o}. \tag{20}$$

2.8 Rent Protection Activities

Adult firms in A industries face the threat of innovation and undertake rent protection activities (RPAs), denoted by X, aiming to prolong the expected duration of temporary monopoly profits by delaying the success of challengers. Adult firms optimally choose X at each point in time to maximize expected returns, as stated in LHS of (17). This maximization yields the following *RPA condition*:[27]

$$w_H \gamma X = I\left(V_a - V_o\right) \qquad \textbf{RPA}. \tag{21}$$

The LHS of (21) equals RPAs expenditure and increases with the threat of innovation I and the capital loss associated with innovation $V_a - V_o$.

2.9 Labor Markets

At each instant in time, each low-skilled worker can either be employed or unemployed. In B industries, when a young firm matches all its vacant positions, an old firm exits the market and fires all of its low-skilled workers. The fraction of industries that experience this type of labor turnover is $q(\theta)(1 - n_A)$. In each B industry the number of workers employed by an old firm is $(1 - \phi)Z$. As a result, the flow of workers into the unemployment pool during time period dt equals $q(\theta)(1 - n_A)(1 - \phi)Zdt$.[28] The flow of workers out of unemployment during time period dt is driven by successful job finding of unemployed workers, which is given by $p(\theta)Udt$. As a result, the equation of motion for the level of unemployment U is given by[29]

$$\dot{U} = q(\theta)(1 - n_A)(1 - \phi)Z - p(\theta)U\ . \tag{22}$$

At each point in time, young technology leaders in B industries maintain vacant positions to hire workers. The fraction of B industries is equal to $1 - n_A$. The number of vacant positions in each industry is equal to labor demand $V_i = (1 - \phi)Z$. Thus, the economy-wide vacancy rate, defined as vacancies per low-skilled worker $v \equiv \sum V_i/(1 - s)N = V/[(1 - s)N]$, equals

$$v = \frac{(1 - n_A)(1 - \phi)Z}{(1 - s)N}\ . \tag{23}$$

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27 The RPA condition is derived as follows. Use (12) and (11), note that $I(X) = R/(\delta X)$, and set the derivative of the LHS of (17) with respect to X to zero. Using $dI(X)/dX = -I/X < 0$ and taking limits as $dt \to 0$ yields (21).

28 There are additional flows into and out of unemployment that cancel out each other. In A industries, with instantaneous probability I, an entrepreneur successfully innovates, the incumbent monopolist loses a fraction ϕ of the market, and lays off ϕZ workers. This event creates an inflow $In_A\phi Z$ into the unemployment pool. However, this is matched by instantaneous hiring of the same number of workers by successful entrepreneurs in all A industries.

29 Note that the equation of motion for vacancies V is the same as (22). In a fraction n_A of industries, with probability I young firms create V_i vacancies. The total matching at each point in time is given by $pU = qV$. It follows that $\dot{V} = In_AV_i - qV$. In the steady-state equilibrium, $\dot{V} = 0$ and therefore $V_i = qV/(In_A)$. Substituting into this expression the stock of vacancies V from equation (23) and taking into account $(1 - n_A)/n_A = I/q$ from in (5) with $\dot{n}_A = 0$, provides $V_i = (1-\phi)Z$, and thus $\dot{V} = In_A(1-\phi)Z - pU$. Using $In_A = (1 - n_A)q$ yields the RHS of (22).

There is a labor market for low-skilled workers and a separate one for high-skilled workers. In each market, the supply of employed workers must equal the demand for labor. As a result, the labor market clearing conditions for low-skilled and high-skilled workers may be written as

$$(1-u)(1-s)N = Z\left[n_A + (1-n_A)\phi + (1-n_A)(1-\phi)\right] = Z\,, \tag{24}$$

$$sN = n_A(\gamma X + \beta R)\,, \tag{25}$$

where $u \equiv U/[(1-s)N]$ is the unemployment rate of low-skilled workers.

Substituting Z from (24) into (22), and using the definitions of u and v, provides the following equation of motion for the rate of unemployment u:

$$\dot{u} = q(\theta)(1-n_A)(1-\phi)(1-u) - p(\theta)u\,, \tag{26}$$

where $q(\theta)(1-n_A)(1-\phi)$ is the economy-wide *job-destruction rate*, and $p(\theta)$ is the economy-wide *job-finding rate*.

3 Steady-State Equilibrium

Based on analytical tractability, we will focus on the steady-state properties of our model where all the endogenous variables are constant over time. Our steady-state analysis might be empirically relevant to explaining movements of unemployment and growth in prolonged recessions. For instance, focusing on a quick adjustment path for unemployment by considering a jump from one steady-state to another is consistent with the empirical evidence provided by Elsby et al. (2013) who find that the US unemployment dynamics are "uncommonly rapid" (ibid, p. 539). They argue that in the context of a matching framework, "the unemployment rate can be approximated very closely by its flow steady-state value" (ibid, p. 539).

In this section we establish that the steady-state equilibrium is unique and analyze its properties. Appendices A, B and C provide algebraic details. We choose low-skilled labor as the numéraire by setting $w_L \equiv 1$.[30] At the steady-state equilibrium, per-capita consumption expenditure c is constant over time. It then follows from equation (4) that the market interest rate is $r = \rho$.[31] In contrast, the arrival of process

[30] Alternatively, one could normalize per capita consumption expenditure by setting $c \equiv 1$. This normalization provides explicit determination of the low-skilled wage rate w_L. Equation (7) implies $Z \equiv cN/(\lambda w_L) = N/(\lambda w_L)$. Combining this result and (24) generates $w_L = 1/[\lambda(1-u)(1-s)]$. This approach, however, complicates the presentation of steady-state equilibrium without providing additional insights.

[31] In addition, the absence of population growth implies that the following endogenous variables remain constant

innovations generates positive endogenous growth of TFP, output, per-capita consumption expenditure measured in units of output, and consumer utility. The arrival of innovations generates deflation as the aggregate price level of final goods falls at constant rate $\dot{P}_{AGG}/P_{AGG} = -n_A I \log\lambda$ (please see Appendix C for details). By imposing $\dot{n}_A = 0$, we obtain the equilibrium share of type A industries as $n_A = q/(I + q)$.

The equilibrium is characterized by the following system of three equations in three unknowns: the matching rate $q(\theta)$; the rate of innovation I, and the rate of unemployment u (please see Appendix A for details). The steady-state values of these variables are constant over time.

$$\frac{q(\theta)}{(1-\phi)}\left[\underbrace{\frac{\lambda-1+\sigma_a}{\rho}-\frac{2\phi\left(\lambda-1+\sigma_y\right)}{B\left(1-\sigma_R\right)\rho^2}}_{V_a/Z}-\underbrace{\frac{\phi\left(\lambda-1+\sigma_y\right)}{\rho}}_{V_y/Z}\right]=\alpha \qquad \textbf{VC}, \tag{27}$$

$$I=\frac{\frac{\phi\left(\lambda-1+\sigma_y\right)}{\rho}}{B\left(1-\sigma_R\right)\left[\underbrace{\frac{\lambda-1+\sigma_a}{\rho}-\frac{2\phi\left(\lambda-1+\sigma_y\right)}{B\left(1-\sigma_R\right)\rho^2}}_{V_a/Z}-\underbrace{\frac{(1-\phi)(\lambda-1)}{\rho+q(\theta)}}_{V_o/Z}\right]} \qquad \textbf{RP}, \tag{28}$$

$$\frac{q(\theta)I}{I+q(\theta)}\left(1-u\right)\left(1-\phi\right)=p(\theta)u \qquad \textbf{BC}. \tag{29}$$

The **vacancy-creation (VC) condition** (27) is the reduced form of (10) and expresses the equilibrium matching rate $q(\theta)$ as a function of parameters. The matching rate is a monotonically decreasing function of labor-market tightness, that is $\partial q(\theta)/\partial\theta < 0$. Let us denote with $\theta \equiv \mu(q)$ the inverse function that determines the market tightness measure θ as a declining function of the matching rate q. Therefore (27) pins down q and (unique) $\theta = v/u$, from which the job-finding rate of workers $p(\theta)$ is determined.

The **relative-profitability (RP) condition** (28) expresses the rate of innovation I as a function of matching rate $q(\theta)$ and parameters. Let us first consider the RHS. Term $1/[B(1-\sigma_R)]$ is the cost of R&D relative to RPAs. The numerator is the market value of a young firm per unit of output $V_y/Z = \phi(\lambda-1+\sigma_y)/\rho$. The term in square brackets captures the expected return to RPAs, measured by the difference in per-output market value between an adult firm and an old firm $(V_a - V_o)/Z$. As a result, the RHS of the RP condition is proportional to $V_y/(V_a - V_o)$ which is the expected return of R&D relative to

over time: rate of vacancies v, R&D investment R, level of rent protection activities X, profit flows, stock market values of firms, allocation of labor across activities, and the wage of high-skilled labor w_H.

RPAs, i.e., the relative profitability of R&D. Observe that the RHS of (28) is monotonically decreasing in q. As a result, once the matching rate q is determined, equation (28) pins down the equilibrium innovation rate I.

The **Beveridge Curve (BC) condition** (29) combines (26) with equation (5), using $n_A = q/(I + q)$. The LHS of (29) corresponds to the rate of labor flow into unemployment, and the RHS represents the rate of labor flow out of unemployment. With q, p and I determined, BC condition (29) and the expression of $\theta = v/u$ from the VC equation (27) determine simultaneously the equilibrium levels of u and v.[32]

We view equation (29) as a general-equilibrium version of the Beveridge curve with endogenous job-destruction. The standard Beveridge Curve in the DMP literature treats the job-destruction rate as a constant, which can be obtained by assuming that the job-destruction function $\Theta(I,\theta) \equiv (1-\phi)q(\theta)I/[I + q(\theta)] = \bar{\Theta}$ is an exogenous parameter. Our model generalizes the DMP theory of unemployment by recognizing explicitly the dependence of the job-destruction rate on labor market tightness θ and the rates of innovation and growth, I and g.[33] More specifically, $\Theta(I, \theta)$ is decreasing in θ and increasing in I, which respond to all general-equilibrium parameters of the model.

We next derive an expression for the rate of growth. The endogenous arrival of process innovations generates growth in instantaneous consumer utility. The latter captures the appropriately weighted consumption index and corresponds to real per-capita income. It is possible to obtain the following expression for instantaneous utility:[34]

$$\log h(t) = \log(c/\lambda) - (1-n_A)(1-\phi)\log\lambda + (\log\lambda)n_A It\ . \qquad (30)$$

The first term captures the effect of per capita output expressed in units of low-skilled labor c/λ under the assumption that all firms charge the same price as young firms. The second term reflects a price adjustment based on the fact that old firms charge a higher price than young firms, $P_o > P_y$. This price prevails in a share $1 - \phi$ of each B industry. These industries account for fraction $1 - n_A$ of all industries. The third term captures the standard dynamic effect caused by the arrival of process innovations: every

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32 With I, q, θ, and u determined, the remaining endogenous variables can be obtained in a standard recursive fashion. The level of n_A follows from imposing $\dot{n}_A = 0$ in (5). Substituting $R = I\delta X$, from (11) and (12), into (25) yields X. Substituting u into (24) gives Z and thereby c, from which π_o follows. π_a and π_y can be recovered from (6) and (9). With π_y and X determined, w_H is derived from (A.2).

33 Recent shifts in the Beveridge curve have been identified and discussed by prominent economists, e.g., Diamond (2011), Mortensen (2011), and Pissarides (2011). Our model offers a Neo-Schumpeterian perspective capturing general equilibrium shifts in the Beveridge curve.

34 Appendix C provides the detailed derivations yielding (30).

time an innovation occurs in an industry, the instantaneous utility jumps up by $\log\lambda$; and during the period from time zero to t the expected number of innovations occurring in each of n_A growth-oriented industries is It. Differentiating (30) with respect to time yields the growth rate of instantaneous utility

$$g = In_A \log\lambda = \frac{Iq\log(\lambda)}{I+q}. \qquad \textbf{GR} \qquad (31)$$

The growth rate is proportional to the rate of innovation I, the measure of growth-oriented industries n_A, and parameter $\log(\lambda)$ capturing the impact of innovation size.

We now illustrate the equilibrium graphically in (q, I) and (u, v) spaces. We assume a Cobb-Douglas matching function: $M(U,V) = AV^{\eta}U^{1-\eta}$, which implies $q = A\theta^{-(1-\eta)}$, $\theta \equiv \mu(q) = \left(q/A\right)^{\frac{-1}{1-\eta}}$, and an elasticity of market tightness θ with respect to the matching rate q denoted by $\varepsilon = -(\partial\mu/\partial q)q/\mu = 1/(1-\eta) > 1$. The VC condition (27), which determines the equilibrium value of q as a function of the model's parameters, is shown in Figure 1a by vertical line VC. The RP condition (28), which establishes an inverse relationship between I and q is shown by the downward sloping curve RP. The intuition behind the shape of curve RP is as follows. A higher matching rate q reduces the valuation of old firms V_o/Z by increasing their replacement rate. Thus, for an adult firm the incentives to avoid replacement and engage in RPAs become stronger. The profitability of RPAs relative to R&D increases, and this leads to a lower innovation rate I.

To show the unemployment and growth rates in (q, I) space, we utilize the BC condition (29) and GR equation (31). The corresponding graphs are illustrated with the iso-unemployment (UU) and iso-growth (GG) curves.

Consider first the UU curve. Totally differentiating (29) for a given u using $q(\theta)\theta = p(\theta)$ and $\theta \equiv \mu(q)$ yields

$$\left.\frac{dI}{dq}\right|_{u=\bar{u}} = -\frac{I}{q}\left(\varepsilon - 1 + \varepsilon\frac{I}{q}\right) < 0, \qquad (32)$$

where $\varepsilon > 1$ under a Cobb-Douglas matching function. In this case, the typical UU curve in (q, I) space is downward sloping and convex to the origin. Moving away from the origin in (q, I) space implies higher unemployment rates. The intuition behind the negative slope of a UU curve is as follows. A higher matching rate q increases aggregate job destruction rate $qI/(I+q)$. Higher q also implies a lower vacancy-unemployment ratio θ and hence a lower job-finding rate p. Both effects raise the unemployment rate u.

Along a UU curve the unemployment rate must remain constant. This requires a reduction in the innovation rate I reducing the labor flow into unemployment by lowering the mass of industries subject to replacement $1 - n_A = I/(I+q)$.

Consider next the iso-growth GG curve. Totally differentiating equation (31) for a given g yields

$$dI/dq\big|_{g=\bar{g}} = -\left(I/q\right)^2. \tag{33}$$

A typical GG curve is convex to the origin and downward sloping. The intuition behind the negative slope is as follows. A higher q increases growth by raising the fraction of growth-oriented industries n_A. Along a GG curve the rate of growth must be constant. This property requires a reduction in the innovation rate I. Moving away from the origin in (q, I) space implies higher growth rates. Direct comparison between (32) and (33) establishes the following property: if $\varepsilon > 1$, a condition which holds under a Cobb-Douglas matching function, then the UU curve is steeper than the GG curve for any pair of (q, I).

Next, we determine the equilibrium levels of u and v by showing the BC and the VC curves in (u, v) space in Figure 1b. The VC curve is shown by a straight line in (u, v) space whose slope equals the equilibrium level of market tightness θ. Note that with q pinned down by the VC condition, the function $\theta \equiv \mu(q)$ determines the equilibrium level of θ. The Beveridge Curve is downward sloping curve in (u, v) space. To see the intuition, consider an increase in v, holding u constant. Restoring the BC requires a lower u for two reasons. First, a higher v reduces the economy-wide matching rate of technology leaders in type B industries $q(\theta)(1 - n_A)(1 - \phi)$, which is also the aggregate *job-destruction rate*.[35] This implies longer tenures for the follower firms in type B industries and hence more job-security for their workers. Consequently, the flow of workers into the unemployment pool decreases. Second, a higher v increases the job-finding rate of unemployed workers $p(\theta)$, and thereby raises the flow of workers out of the unemployment pool. To sum up, as the vacancy rate v increases, more workers escape unemployment and fewer workers join the jobless; thus, the unemployment rate u decreases to restore equilibrium.

Insert here: Figure 1a-b (Steady-state equilibrium)

Figure 1a illustrates the iso-unemployment UU and iso-growth GG curves passing through the initial equilibrium levels of u and g. These curves divide the (q, I) space into four quadrants. It follows from (29) and (31) that the GG curve depends only on λ, and that the UU curve depends only on technological and

[35] Note that $q(\theta)(1 - n_A) = q(\theta)I/[I + q(\theta)]$. Thus, one can show that $\partial[q(\theta)(1 - n_A)]/\partial v < 0$.

policy-invariant parameters ϕ and ε. The rest of parameters, including policy-related ones that change the initial equilibrium by shifting the RP and the VC curves, neither affect the position nor the shape of iso-unemployment and iso-growth contours. This leads to the following result.

Proposition 1: *An economic policy that shifts the initial equilibrium point E in Figure 1a to a new equilibrium point located in:*

- *quadrant I generates higher growth g and lower unemployment rate u;*
- *quadrant II generates higher growth g and higher unemployment u;*
- *quadrant III generates lower growth g and higher unemployment rate u;*
- *quadrant IV generates lower growth g and lower unemployment u.*

The four quadrants in Figure 1a illustrate clearly the *conundrum* associated with recovery policies or economic shocks. Quadrants II and IV show the area where growth and unemployment are positively correlated. A move from the initial equilibrium to any point in quadrant II generates jobless growth; whereas a move to any point in quadrant IV results in stagnant growth with job creation. One of the main insights of the neo-Schumpeterian approach to macroeconomics is that higher growth may *not* necessarily come with lower unemployment and vice versa. Quadrants I and III establish a negative relationship between unemployment and growth as one would find in a conventional textbook model of macroeconomics. A move to a final equilibrium point located in quadrant I generates an economic recovery with higher output growth and lower unemployment; whereas a move to quadrant III generates a recession with lower growth and higher unemployment.

4 Prolonged Recessions and Recovery Policies

We begin our analysis by considering shocks that can lead to low growth and high unemployment (prolonged recessions). Then we analyze the effects of possible recovery policies on unemployment and growth. To resolve ambiguities, we perform numerical simulations using benchmark parameters from the U.S..

Insert here: Table 1 (Numerical analysis)

Table 1 shows the numerical implementation of our model for the benchmark case, as well as numerical results that correspond to the policy analysis in subsections 4.3 – 4.4. We chose the size of innovations, $\lambda = 1.25$, so as to be consistent with the gross markup (the ratio of the price to the marginal cost) enjoyed by innovators. According to the literature, the value of the markup is between 1.05 and 1.4 (see Basu 1996 and Norrbin 1993). The subjective discount rate ρ is set at 0.05 capturing a real interest rate of 5 percent. Jones and Williams (2000, p. 73) argue in favor of using such relatively high real interest rates rather than risk-free rates on treasury bills of around 1%. In the DMP simulation literature, Hall (2005) and Shimer (2005) use 5%, whereas Felbermayr and Prat (2011) use 4%. The matching function takes the Cobb Douglas form as in Blanchard and Diamond (1989) with $M(U, V) \equiv AV^{\eta}U^{1-\eta}$ where we assumed $\eta = 0.6$, such that $q = A(1/\theta)^{0.4}$ and $p = A\theta^{0.6}$. Our choice of $\eta = 0.6$ is in line with the DMP simulation literature. Hall (2005) estimates η as 0.76. Shimer (2005) uses $\eta = 0.28$. Felbermayr and Pratt (2011) use $\eta = 0.5$. Lubik (2013) and Kitao et al. (2011) use values of 0.42 and 0.5, respectively. We set $\gamma = 1$, which is an inconsequential parameter that affects only the level of RPA activity per capita, x. We set the proportion of high-skilled workers s at 0.05 to generate a wage differential $w_H/w_L \equiv w_H$ that is significantly greater than 1. Our definition of "high-skilled" workers is very narrow, because it comprises only those working in R&D and RPAs. $N = 1$ and $w_L = 1$ are convenient normalizations. Finally, setting $\sigma_a = \sigma_y = \sigma_R = 0$ serves as a useful, distortion-free reference case.

We treat ϕ, A, $B \equiv \beta\delta/\gamma$ and α as free parameters and choose their levels with the objective of attaining the following targets: $v = 0.03$, $u = 0.06$, $g = 1.5\%$, $n_A = 0.965$. To attain these targets, we need to set $A = 1.455$, $B = 4.904$, $\phi = 0.0881$,[36] and $\alpha = 2.03$. We target an unemployment rate of 0.06, which is in line with the long-term US unemployment rate of 6.1% (calculated from the FRED database for the period 1960-2012). We target an annual growth rate of 1.5%, which is in line with the papers that use numerical simulations in growth-based models. Gustafsson and Segerstrom (2011) use a growth rate of 2%. Chu et al. (2012) and also Davis and Şener (2012) use 1.5%. The long term US GDP per-capita growth rate is 2.0 % for the period 1960-2011, and 1.3% for the period 1990-2011 (calculated again from FRED database). We target a vacancy rate of 3%, which is in line with the US average of 2.8% for the 2003-2012 period. The data is reported by the BLS under the JOLTS (Job Openings and Labor Turnover Survey) program.[37] Finally, we target a reasonable level for n_A that is consistent with an interior equilibrium given the other targets and parameters. Our high target of $n_A = 0.965$ implies that, at any given time, innovation is taking place in almost all industries, an outcome which we think bodes well with the real world.

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[36] We note that a low ϕ corresponds to a high degree of labor market frictions faced by successful innovators. It also implies *ceteris-paribus* a low R&D reward. Initial low profits earned by young successful firms are consistent with the notion of "crossing the chasm" in studies of how high-tech markets evolve over time, e.g., see Moore (2002).

[37] We should note that the exact vacancy rate used in the JOLTS is slightly different from our definition $v \equiv V/[(1-s)N]$. The BLS calculates v_{JOLTS} by taking the ratio of vacancies to the sum of employment and vacancies. In our model, this corresponds to $v_{JOLTS} = \frac{v}{1-u+v}$. Using the benchmark simulation results of Table 1, we can calculate v_{JOLTS} as 0.0309, which is in line with the vacancy rate of 2.8% as reported in the JOLTS.

We start our analysis by considering shocks that can lead to lower growth and higher unemployment. We use Figure 1a to illustrate the "conundrum of recovery policies" and the growth and unemployment effects of various policies. In Figure 1a, we draw the RP curve such that it lies between the UU and GG curves. This turns out to be the empirically relevant case which is consistent with numerical simulations.

4.1 Adverse shocks to profit flows

Adverse shocks to profit flows triggered by weak aggregate economic activity has been emphasized by Pissarides (2011, p. 1097) and Mortensen (2011, p. 1087) in the context of search and matching models that analyze the Great Recession. In our model, an analogous mechanism that leads to lower profit flows can be initiated by a decline in innovation size λ, which lowers mark-up rates and thus profit flows. We also think that shocks to λ can be interpreted as TFP growth shocks since other policy or technology parameters (affecting g through the innovation rate I) seem more stable over time. Moreover, shocks to λ can capture input price shocks that affect profit margins. We now focus on the specific mechanisms implied by our model. We consider the benchmark case with no subsidies. In this case, a lower λ exerts no influence on the RP condition.[38] It shifts the VC curve to the right if and only if $B\rho(1-\sigma_R)(1-\phi)-2\phi > 0$, a parametric restriction which clearly holds in our simulations. Consequently, the rate of innovation I decreases, and the matching rate q increases. In Figure 1a, the economy moves to quadrant 3, with higher unemployment and lower growth.

What is the intuition? For a given matching rate, a common adverse shock to profit flows reduces all firm valuations V_a, V_y and V_o in the same exact proportion, leaving the ratio $V_y/(V_a-V_o)$ intact and hence the relative R&D-RPA incentives. However, a lower λ reduces the valuation differential between adult and young firms, V_a-V_y and generates a fall in the marginal profitability of vacancy creation. This leads to fewer vacancies opened up, decreasing the number of vacancies per unemployed $\theta \equiv v/u$ ratio. The lower θ facilitates the search efforts of young firms and implies an increase in their matching rate q. Formally, the rise in q and the fall in θ follow from (27) and $\theta \equiv \mu(q)$, respectively. The impact on the innovation rate I follows from the downward sloping RP curve. The higher matching rate q reduces the valuation of old firms V_o by increasing their replacement rate. Adult firms have now more incentives to avoid becoming an old firm and defend their positions. This translates into a fall in the relative R&D-

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[38] In the more general case of positive subsidies, one could easily show that in response to a lower λ the RP curve shifts to the left iff $q(\sigma_a-\sigma_y)+\rho(\sigma_a-\sigma_y\phi)>0$. Numerical simulations show that the RP curve can indeed shift to the right and $dI/d\lambda < 0$, but when we confine the simulations to reasonable levels of u, by which we consider $u < 0.3$, the negative relationship between u and g triggered by λ survives.

RPA incentives and reduces the innovation rate I.

We focus on the (u, v) space to examine the changes in unemployment and vacancy rates, as illustrated in Figure 2a. As in the standard DMP model, reduced profit flows weaken vacancy creation incentives and leads to a lower $\theta \equiv v/u$ ratio. This translates to a clockwise turn of the VC curve, which puts upward pressure on unemployment and downward pressure on vacancies. In addition, in our model the lower innovation rate reduces the fraction of B industries and thus decreases the job-destruction rate. For a given level of vacancies, this puts downward pressure on unemployment. The resulting inward shift of the BC curve mitigates the rise in unemployment but reinforces the fall in vacancies. Thus, the vacancy rate v unambiguously declines, and numerical simulations imply that the unemployment rate u increases.

With regards to growth, we identify three competing effects. First, the lower rate of innovation I reduces growth. Second, the higher fraction of growth-oriented industries n_A driven by higher q and lower I increases growth. Finally, the direct fall in innovation size λ reduces growth. The net impact points to a fall in the growth rate g.

Insert here: Figure 2a-f (*u*-*v* simulations)

Proposition 2: *An adverse shock in the form of a reduction in innovation size λ increases the matching rate of young firms q. For the empirically relevant case where the RP curve is between the UU and GG curves, the economy moves to quadrant* III, *attaining a lower rate of growth g, a lower rate of innovation I, and a higher rate of unemployment u.*

To assess the quantitative relevance of the model we consider a negative shock to λ that will increase the unemployment rate to 0.10, which is roughly equal to the peak unemployment rate in the US during the Great Recession. As shown in Table 1, numerical simulations indicate that a decline in λ from 1.25 to 1.17, a 6% decline, indeed raises the unemployment rate from 0.06 to 0.10 and implies an instant decline in total output by around 4%.[39] The vacancy rate v also declines from the benchmark level of 0.03 to 0.0192, which is roughly in line with the US vacancy rate of 1.8% observed at the recent peak unemployment rate of 9.9% in October 2009, as reported by JOLTS. The labor market tightness ratio $\theta \equiv v/\mathrm{u}$ falls from 0.5 to around 0.2. The subsequent TFP growth rate g is lower, declining from the benchmark level of 0.015 to 0.01. Thus our model appears capable of generating large movements in unemployment, vacancies, the tightness ratio θ, as well as output levels and growth. In this context, our

[39] To see this, note that total output Z is proportional to employment rate $(1 - u)$ by equations (7) and (24). All firm valuations are in turn proportional to Z.

model provides a response to the Shimer (2005) critique, according to which the standard DMP model cannot generate the observed fluctuations in the v/u ratio in response to plausible shocks. Figure 2a shows the resulting shifts in the (u, v) space. The shift in the VC curve is substantial, whereas the mitigating inward shift in BC is relatively modest. Hence, the rise in u and the fall in v are mostly driven by changes in job-creation incentives amidst a stable BC curve.

At the new equilibrium, the economy allocates a larger portion of its high-skilled labor force to RPAs, as measured by $n_A X\gamma$, which increases from 3.72% to 3.75%. The stock market valuations for all firms decline, with the total market valuation V^{TOT} falling from the benchmark level of 1.2158 to 0.799, a 34.3% decline! The drop in stock market is primarily triggered by three forces. First is the lower mark-up rate, which directly reduces profit flows. Second is the higher unemployment rate, which indirectly reduces total sales volume.[40] These two effects reduce valuations for all firms. Third, there is a particular effect on old firms' valuations. The rise in the matching rate of young firms q implies a higher turnover rate for old firms, reducing their valuations V_o. This is why old firms suffer the largest decline in their valuations by around 55.4%. In comparison, young and adult firms experience a decline of 34.9%.

Our model establishes a new channel through which adverse profit shocks affect unemployment. First, as in the standard DMP models, such shocks weaken vacancy-creation incentives and increase unemployment by shifting the VC curve clockwise. In addition, this type of an adverse shock generates a *self-mitigating effect* on unemployment by reducing the innovation rate and thus the job-destruction rate. This effect, which mitigates but does not overturn the rise in unemployment, is captured by the inward shift of the BC curve. The same exact mechanism is triggered in the opposite direction in response to a positive shock to profit flows. In addition, our result that a lower innovation size reduces growth but raises unemployment is in sharp contrast to the findings of Aghion and Howitt (1994, p. 488). In their endogenous growth setting, Aghion and Howitt find that a lower innovation size necessarily reduces both unemployment and growth.[41]

4.2 Reduced matching efficiency

We now consider another adverse shock that specifically affects the labor market: a reduction in matching

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[40] To see these, observe that reduced form valuations stated in equations (27) and (28), which are expressed as being divided by Z, where $Z = (1 - u)(1 - s)N$ from equation (24). It should be noted the impact of innovation washes out in these valuations once the optimal RPA levels of firms are substituted. See Appendix A for further details.

[41] From an empirical viewpoint, our finding that shocks to λ generate a negative correlation between TFP growth and unemployment is consistent with the short-run evidence as shown by Postel-Vinay (2002, Figure 1 on p. 740).

efficiency captured by a decline in parameter A. In equation (27) the valuation differential that determines the returns from vacancy creation, $(V_a - V_y)/Z$ remains unresponsive to shocks to A. Thus, the matching rate q remains the same. Given that $q = A\theta^{-(1-\eta)}$, the labor market tightness ratio $\theta = v/u$ falls sufficiently to return the matching rate $q(\theta)$ to its initial level. The intuition can be seen by substituting $q = A\theta^{-(1-\eta)}$ into (27). A reduction in the efficiency of young firms' search efforts renders matching less profitable. Maintaining the VC condition requires that fewer vacancies be opened per unemployed worker. It follows from (27) and (28) that the VC and RP curves remain unaffected. Hence, the innovation rate I and therefore the growth rate g also remain the same.

To determine the changes in u and v, we focus on the BC and JC curves in (u, v) space as illustrated in Figure 2b. The lower $\theta = v/u$ ratio implies a clockwise turn of the VC curve in the (u, v) space. The fall in matching efficiency A also shifts the BC curve outward. The reason is that a lower A reduces the job finding rate p for any given vacancy rate v, reducing the outflow from the unemployment pool and raising unemployment. The unemployment rate u unambiguously increases. Numerical simulations show that the vacancy rate v declines.

Proposition 3: *A reduction in matching efficiency A does not affect the matching rate of young firms q, the rate of growth g, and the rate of innovation I, while increasing the rate of unemployment u.*

To assess the quantitative impact of this exercise, we generate an adverse shock to A leading to an increase in unemployment rate u from 0.06 to 0.10. This is attained by a fall in A from 1.4555 to 1.16, corresponding to a 20% decline, which reduces v from 0.03 to 0.287. The simulated graph in Figure 2b indicates that shifts in both the VC and BC curves contribute to the rise in unemployment. With matching rate of young firms q constant, the higher unemployment leads to a relatively modest 4.3% decline in total stock market valuation V^{TOT}. The share of high-skilled allocated to RPA $nA\gamma X$ remains almost constant.

Recovery Policies

We now consider possible policies to reduce unemployment and promote growth, which in general are the two objectives that are explicitly stated by the policy makers who face a recession.[42]

[42] An optimal policy analysis that focuses on long-run welfare is left out due to space considerations. This is available from the authors upon request.

4.3 Policies aimed at reducing vacancy costs

Vacancy costs may consist of advertising vacant positions, maintaining a human resources department, reimbursing outlays of applicants (e.g. travel and lodging costs), etc.[43] In Figure 1a, a decrease in vacancy cost parameter α shifts the VC curve to the left without affecting the RP curve. The innovation rate I increases, the matching rate q declines and the v/u ratio increases. Under the empirically relevant case, where the RP curve lies between the UU and GG curves, the economy moves to quadrant I where growth is higher and unemployment is lower.

What is the intuition behind these desirable effects? A lower vacancy cost α strengthens the vacancy creation incentives and leads to more vacancies per unemployed worker $\theta \equiv v/u$. The higher θ makes it more difficult for young firms to match their positions, thus the matching rate q decreases. A lower matching rate q increases the valuation of old firms V_o because their likelihood of replacement decreases. As a result, adult firms have weaker incentives to defend their positions through RPAs. This translates into a rise in the relative R&D-RPA profitability, leading to a higher innovation rate I. These mechanisms show that RPAs play a key role in generating the innovation response. Observe that a change in α does not directly affect R&D and RPA incentives. Nevertheless, it indirectly weakens the incentive to conduct RPAs relative to R&D.

To determine the changes in u and v, we focus on Figure 1b. The rise in the $\theta \equiv v/u$ ratio turns the VC curve counterclockwise, reducing u and increasing v. The increase in the innovation rate and thus job-destruction rate triggers an outward shift of the BC curve, while the decrease in q does the opposite. Numerical simulations imply that the BC curve shifts only slightly outward (see Figure 2c), such that v increases and u decreases. Growth is affected through two channels: higher innovation rate I promotes growth, whereas the reduction in the fraction of growth-oriented industries n_A (driven by lower q and higher I) reduces the rate of growth. The former effect dominates, increasing the equilibrium rate of growth g.

The aforementioned analysis leads to

Proposition 4: *Policies that reduce the costs of vacancies α reduce the matching rate q. For the empirically relevant case where the RP curve is between the UU and GG curves, the economy*

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[43] A decrease in α may become operational by defining marginal vacancy creation costs $\alpha \equiv \hat{\alpha}(1-\sigma_V)$, and considering an increase in the vacancy creation and maintenance subsidy rate σ_V. See Hagedorn and Manovskii (2008, pp. 1698-99) and the references therein for empirical evidence on search costs associated with vacancies.

moves to quadrant I, *attaining a higher rate of growth g, a higher rate of innovation I, and a lower rate of unemployment u.*

To investigate the quantitative impact of this shock, we consider an arbitrary decline in α, from 2.03 to 1.80, roughly amounting to a 10% fall. This reduces u from 0.06 to 0.0508 and increases g from 0.015 to 0.0151. Even though the unemployment rate goes down by an almost full one percentage point, the rise in growth is very modest. The vacancy rate v increases substantially though to 3.44%. The numerical representation in (u, v) space of Figure 2c shows a sizeable counterclockwise turn in the VC curve with an almost stable BC curve. The stock market valuation V^{TOT} increases only by about 0.5 %. The share of resources allocated to RPAs goes down modestly from 0.0373 to 0.0372.

4.4 Production and R&D Subsidies

a. Production Subsidies Targeting Small Young Firms

An increase in production subsidy for young, small and scalable (job-creating) firms σ_y reduces the marginal profitability of vacancy creation by increasing the market valuation of young firms V_y and reducing the market value of adult firms V_a, as indicated by the LHS of (27). In Figure 1a, the VC curve shifts to the right, leading to a higher matching rate q and lower $\theta \equiv v/u$ ratio.[44] An increase in σ_y raises also the returns to R&D relative to RPAs, captured by the RHS of (28). For a given matching rate q, higher R&D profitability stimulates innovative activity I and hence the RP curve also shifts to the right. In Figure 1a, with both RP and VC curves shifting to the right, the economy can potentially move to quadrant II or III. Clearly, q increases but the change in I is indeterminate. Numerical simulations show that the rate of innovation I increases.

In regards to economic intuition, note that several competing forces affect the rate of innovation. A larger σ_y directly increases the valuation of young firms V_y, strengthening the entrepreneurs' incentives to engage in R&D. However, there exist indirect effects that work through the higher q and the lower V_a, which are identified above. The higher matching rate q reduces the valuation of old firms V_o and encourages adult firms to increase RPAs. This indirect effect reduces the relative R&D-RPA profitability

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[44] The mechanism behind this result is as follows. An increase in σ_y raises the profit flows of young firms π_y in (9), and thus their valuation V_y in (16). This also implies higher rewards from innovation as captured by the V_y term in (14). Maintaining the R&D free-entry condition (14) requires an increase in R&D costs, which are proportional to $w_H\gamma X$. This, in turn, translates into higher RPA expenditure for adult monopolists in A industries and thus their valuation V_a as captured in (18) decreases. With the marginal gains from vacancy creation $V_a - V_y$ decreasing, each young firm in B industries creates fewer vacancies. In other words, the market tightness $\theta \equiv v/u$ decreases and $q(\theta)$ rises, an adjustment which restores the zero-profit condition in vacancy creation (10).

and exerts downward pressure on I. The lower valuation of adult firms V_a works in the opposite direction by reducing their RPA incentives. Numerical simulations show that the direct effect dominates, increasing the equilibrium rate of innovation I.

In the (u, v) space of Figure 1b, the weaker job creation incentives and the resulting fall in $\theta \equiv v/u$ ratio translates into an clockwise turn of the VC curve. The increase in the innovation rate I and the resulting increase in the job-destruction rate implies an outward shift of the BC curve. The unemployment rate unambiguously increases. Numerical simulations imply that the vacancy rate also increases. With regards to growth, we observe two effects working in the same direction. The faster innovation rate I and the increase in the fraction of growth-oriented industries n_A driven by higher q (despite the mitigating effect of higher I) both work to raise the growth rate g.

In short, a subsidy to small young firms creates a tradeoff between jobs and growth and hence a recovery-policy conundrum. It increases the rates of innovation and growth by increasing the relative profitability of R&D. It also reduces the profitability of vacancy creation by encouraging firms to remain small and young. Thus a subsidy to young firms decreases the job-finding rate of workers and increases the replacement rate of old firms. These two effects along with the rise in the mass of B industries subject to turnover lead to more unemployment.

Numerical simulations show that subsidies to young firms σ_y can lead to substantial responses in the endogenous variables. Starting from the no-subsidy benchmark, setting $\sigma_y = 0.02$ raises the unemployment rate u from 0.06 to 0.1365 and raises the growth rate g from 0.015 to 0.0202. In Figure 2d, both the VC and BC curves turn / shift to the right in a substantial fashion. The vacancy rate goes down from 0.03 to 0.0247. The total stock market value V^{TOT} decreases by about 26%. Note that the level of RPAs at the industry level X decreases by 8.29% and the share of resources allocated to RPAs $n_A \gamma X$ decreases from 0.0373 to 0.0343. These substantial declines highlight the role of RPA response in generating large movements in the growth rate.

b. Production Subsidies Targeting Large Adult Firms

An increase in σ_a raises the marginal profitability of vacancy creation by raising the valuation of adult firms V_a, as captured by the LHS of (27). In Figure 1a, the VC curve shifts left leading to a lower matching rate q and higher $\theta \equiv v/u$ ratio. An increase in σ_a also reduces the relative profitability of R&D as indicated by the RHS of (28). For a given q, this hinders innovative activity I and hence the RP curve also shifts left. In Figure 1a, with both RP and VC curves shifting left, the economy can potentially move to quadrant I or IV. Clearly, q decreases but the change in I is indeterminate. Numerical simulations show that the equilibrium rate of innovation I decreases.

What is the intuition? A higher subsidy to adult firms σ_a directly increases their valuation V_a by increasing their profit flows. Why should this encourage the matching effort of young firms? The reason is that young firms make their vacancy-creation decisions by taking into account the increase in market value upon successful matching. Subsidies to adult firms raise their market value and strengthen the incentives of young firms to engage in vacancy creation in order to become adult firms. Young firms open up more vacancies generating a higher v/u ratio, and a lower matching rate q.

What changes the innovation rate despite the fact that for given unemployment rate u, there is no direct effect of this subsidy on the profit margins of young firms and thus on their valuation V_y? The mechanism again works through RPA incentives and relative R&D profitability. Specifically, when adult firms are subsidized and attain higher valuations, they now have higher incentives to defend their position through RPAs. We should note that there is a mitigating factor due the lower matching rate q, which increases the valuation of old firms V_o and reduces the RPA incentives of adult firms. The net effect is a reduction in the relative R&D-RPA profitability and a decline in the equilibrium rate of innovation I.

We again use Figure 1b to analyze the effects of a higher σ_a on u and v. First, the rise in $\theta \equiv v/u$ ratio turns the VC curve counterclockwise, reducing u and increasing v. Second, the lower innovation rate I shifts the BC curve inward, reducing both u and v. Thus, the stronger job-creation incentives and the lower job-destruction rate both work to reduce the unemployment rate u. Numerical simulations show that the vacancy rate v increases.

In short, a policy of subsidizing large adult firms implies a tradeoff between growth and jobs. It belongs to the conundrum of recovery policies. It raises the returns from vacancy creation by rewarding successful job-matching more generously. This helps with fighting unemployment. It has no direct effect on R&D incentives (as measured by the direct rewards from successful R&D V_y) but it motivates adult firms to defend their positions more rigorously. This reduces the relative R&D-RPA profitability and retards innovation. The lower innovation rate helps to further reduce unemployment by slowing the rate of industry turnover.

Numerical simulations show that raising the subsidy rate for adult firms from zero to 0.02 reduces the unemployment rate u from 0.06 to 0.0288, but also reduces the growth rate g from 0.015 to 0.0117. Both effects are quite large. In the (u, v) space of Figure 2e, the shifts / turns in BC and VC curves are also sizable. The vacancy rate increases from 0.03 to 0.0343. The stock market valuation V^{TOT} increases by a substantial 32.3%. The decline in growth is accompanied by a considerable increase in RPAs by adult firms. The level of RPAs at the industry level X increases by 6.22% and the RPA share of resources increases from 0.0373 to 0.0394.

c. R&D Subsidies

An increase in R&D subsidy rate $\sigma_R > 0$ reduces the market valuation of adult firms V_a in (27) and (28).[45] As a result, this growth-oriented policy generates the same qualitative effects as those triggered by an increase in production subsidy targeting young firms σ_y. It increases the rates of growth and innovation by reducing the costs of the latter, and raises the rate of unemployment by leading to a higher labor turnover. It thus creates a recovery-policy conundrum as well.

Numerical simulations show that setting the subsidy rate for R&D firms at 0.02 increases the unemployment rate u from 0.06 to 0.0717 and also the growth rate g from 0.015 to 0.0161. The vacancy rate v decreases from 0.03 to 0.0294. The resulting shifts are shown in Figure 2f. The stock market valuation V^{TOT} decreases by 6.37%. The level of RPAs at the industry level X decreases by 1.81% and the RPA share of resources $n_A \gamma X$ decreases from 0.373 to 0.0366.

The analysis of production and R&D subsidies leads to

Proposition 5: *The following policies raise the matching rate q, and (based on numerical simulations) increase the rates of unemployment u, growth g and innovation I:*

i) *an increase in young firms' production subsidy rate* $\sigma_y > 0$;

ii) *a decrease in adult firms' production subsidy rate* $\sigma_a > 0$;

iii) *an increase in the R&D subsidy rate* $\sigma_R > 0$.

The preceding discussion sheds light on how policy responses to a recession can affect growth, unemployment and vacancies, in a unified framework. Our model shows that production subsides that are targeted at young firms (as defined in our model) have an adverse employment effect even though they stimulate growth. Likewise, policies that are targeted at adult firms (as defined in our model) indeed have favorable employment but adverse growth effects. We should note that the policy response of the US to the Great Recession indeed involved a number of policies that fall under both categories defined above, which are in most of cases promoted as raising *both* employment and growth. Our model could be of particular relevance by highlighting the tradeoff between employment and growth effects.

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[45] An increase in the tax rate on RPAs could be incorporated by replacing B in (28) with $\hat{B} \equiv B/(1 + \sigma_X)$ and considering an increase in $\sigma_X > 0$. This policy is isomorphic to an increase in an R&D subsidy σ_R because it reduces the relative profitability of RPAs.

5 Welfare

Substituting expression (30) into equation (1) and evaluating the resulting integral yield the following expression for the household steady-state discounted utility:

$$H = \frac{1}{\rho}\left[\frac{n_A I \log(\lambda)}{\rho} + \log\left(\frac{c}{\lambda}\right) - (1-n_A)(1-\phi)\log(\lambda)\right].$$

Note that $g = n_A I\log(\lambda)$ and $c/\lambda = (1 - u)(1 - s)$, which follows from (24) and (7). Thus, policies that increase g (dynamic effect) and reduce unemployment u (a static effect) are good candidates to raise steady-state welfare. Nevertheless, we also need to consider the change in $1 - n_A$, which captures the presence of old firms in B industries charging higher prices (a static effect).

In general the welfare effects of policies are ambiguous and depend on parameter values. It is apparent from the welfare expression that policies generating a recovery conundrum yield ambiguous welfare effects. However, even policies that increase growth and reduce unemployment may generate ambiguous welfare effects. For example, consider a reduction in vacancy costs α that increases g and reduces u. Both effects increase welfare. However, a lower α reduces q and increases I leading to an increase in $1 - n_A = I/(I + q)$ and thus a decrease in welfare. As a result, the overall welfare effect of lower vacancy costs is in principle ambiguous but turns out to be positive for our parameter choices in Table 1.

6 Concluding Remarks

This paper is a first to adopt a neo-Schumpeterian macroeconomic approach to growth and jobs. It develops a model that highlights the general-equilibrium nexus among unemployment and fully-endogenous Schumpeterian growth. Unemployment is modeled according to the DMP theory. Fully-endogenous growth stems from the market interaction between profit-maximizing R&D efforts of entrepreneurs and rent protection activities (RPAs) of adult firms that wish to protect the flow of temporary profits. RPAs deliver a scale-free growth environment, but they set up the conundrum of recovery: policies that reduce the rate of unemployment may reduce the rate of growth by increasing the profitability of RPAs relative to R&D investment, and by shifting resources away from firms engaged in R&D.

The model delivers a steady-state equilibrium which is unique. It also generates a version of the Beveridge curve that allows us to trace the general-equilibrium effects of recovery policies on unemployment and vacancies. We use it to analyze the effects of several recovery policies aiming at reducing unemployment or/and accelerating economic growth. Table 2 provides a summary of analytical and numerical results.

Insert here: Table 2 (Summary of results)

Growth-stimulating policies, such as R&D subsidies or production subsidies targeting young firms searching for workers, have trade-offs. They indeed boost growth but also raise unemployment. In contrast, trade-offs between growth and employment disappear with certain policies. For example, subsidizing the costs of vacancy creation directly (i.e., by reducing vacancy creation costs during the search process) results in higher growth *and* higher employment.

We are the first to admit that these novel results are suggestive rather than conclusive: they depend on reasonable but undoubtedly restrictive assumptions; and in some cases on numerical simulations. For instance, the model ignores transitional dynamics; omits human and physical capital accumulation; and the scale effect property is removed in a particular way. The model assumes that subsidies are financed by lump sum taxes and abstracts from public finance issues stemming from government budget deficits. Relaxing these assumptions leads to feasible and welcome generalizations and extensions of our model.

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Table 1: Simulation Analysis

Benchmark parameters: $\lambda = 1.25$, $\rho = 0.05$, $\alpha = 2.0324$, $\phi = 0.0881$, $N = 1$, $s = 0.05$, $A = 1.4555$, $\eta = 0.6$, $\sigma_y = 0$, $\sigma_a = 0$, $\sigma_R = 0$, $B \equiv \beta\delta/\gamma = 4.9047$, $\gamma = 1$, $w_L = 1$.

Endogenous variables	**Benchmark**	λ **= 1.17**	***A*=1.16**	α **= 1.80**	σ_y **= 0.02**	σ_a **= 0.02**	σ_R **= 0.02**
Innovation rate I	0.0697	0.0677	0.0697	0.0705	0.0933	0.0547	0.0748
Unemployment rate u	0.0600	0.1007	0.1012	0.0508	0.1365	0.0288	0.0717
A-industry share n_A	0.9650	0.9766	0.9650	0.9602	0.9687	0.9613	0.9652
Consumption exp. c	1.1163	0.9995	1.0673	1.1272	1.0254	1.1533	1.1024
Growth rate $g = n_A I \log\lambda$	0.0150	0.0104	0.0150	0.0151	0.0202	0.0117	0.0161
Vacancy rate v	0.0300	0.0192	0.0287	0.0344	0.0247	0.0343	0.0294
Tightness θ	0.5000	0.1907	0.2835	0.6773	0.1806	1.1899	0.4103
RPA level X	0.0386	0.0384	0.0386	0.0387	0.0354	0.0410	0.0379
High-skilled wage w_H	2.0778	1.3583	1.9868	2.0938	2.2480	2.0213	2.1341
Job finding rate p	0.9603	0.5385	0.5445	1.1522	0.5213	1.6156	0.8529
Matching rate q	1.9206	2.8244	1.9206	1.7010	2.8861	1.3578	2.0786
Firm value V_a	1.2552	0.8166	1.2003	1.2675	0.9172	1.6660	1.1749
Firm value V_y	0.3936	0.2560	0.3763	0.3974	0.3905	0.4066	0.3887
Firm value V_o	0.1033	0.0461	0.0988	0.1174	0.0637	0.1614	0.0944
Stock market value V^{TOT}	1.2158	0.7990	1.1626	1.2227	0.8914	1.6081	1.1383
Welfare	3.5942	0.9383	2.6981	3.8074	3.9768	2.9227	3.7926
RPA Share ($n_A \gamma X$)	0.0373	0.0375	0.0373	0.0372	0.0343	0.0394	0.0366

Notes: Here we provide the main results of a Mathematica© Appendix, which is available upon request and also on the authors' websites. The stock market value V^{TOT} is defined as $V^{TOT} \equiv n_A V_a + (1 - n_A)[\phi V_y + (1 - \phi)V_o]$.

Figure 1a: Steady-State Equilibrium in (*I*, *q*) Space
RP: R&D-RPA Relative Profitability, VC: Vacancy Creation, GG: iso growth, UU: iso unemployment

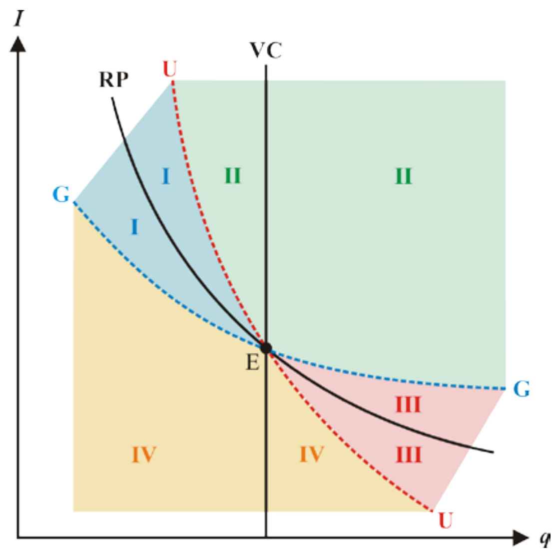

Figure 1b: Steady-State Equilibrium in (*u*, *v*) Space
BC: Beveridge Curve, VC: Vacancy Creation Condition

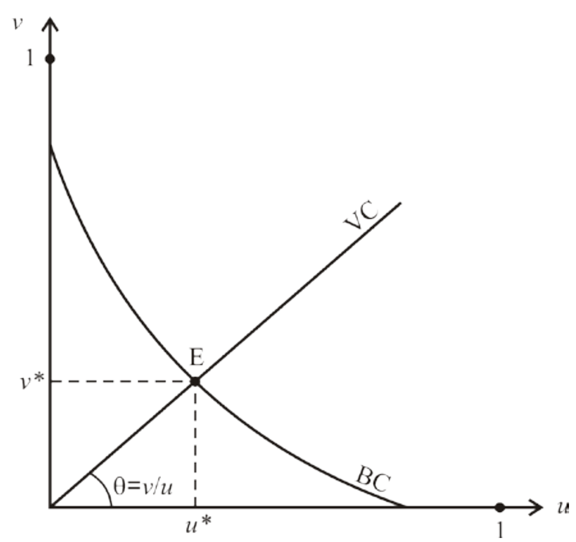

Figure 2a: λ =1.17

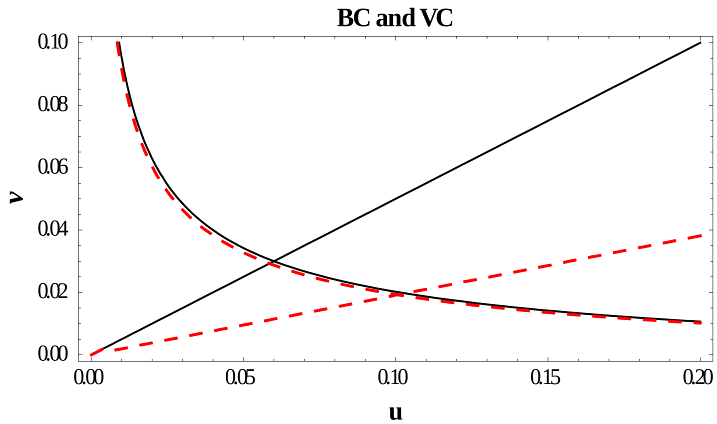

Figure 2b: A= 1.16

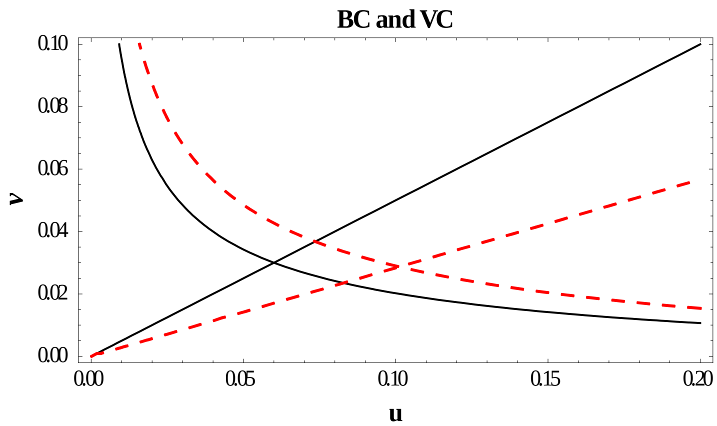

Figure 2c: α =1.90

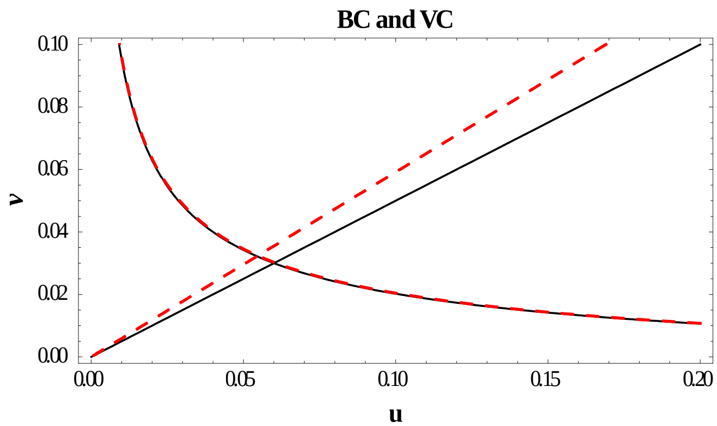

Figure 2d: σ_y **=0.02**

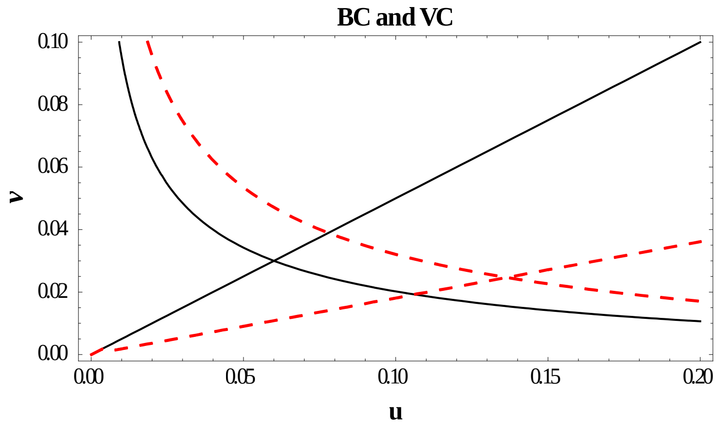

Figure 2e: σ_a **=0.02**

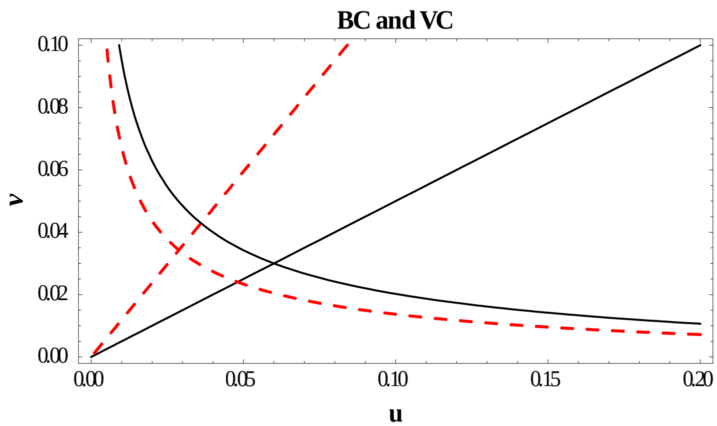

Figure 2f: σ_R **=0.02**

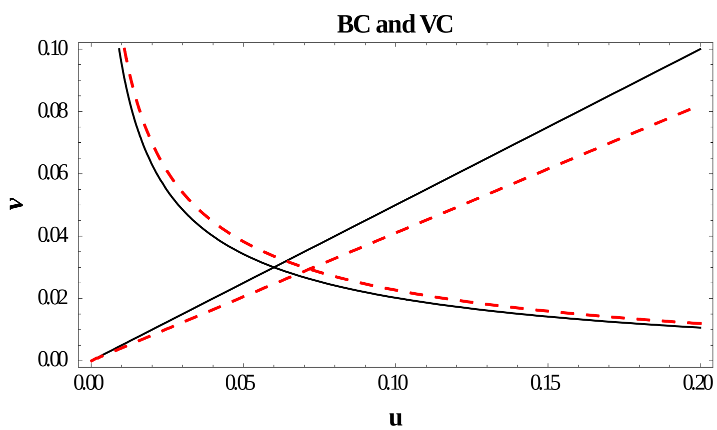

Table 2: Summary of Results

Parameter change	Effects in Figure 1a			Analytical results						Simulation results					
	VC	**RP**	**Quadrant move**	q	I	g	u	v	n_A	q	I	g	u	v	n_A
σ_a ↑	left	left	I, IV	↓	↑↓	↑↓	↓	↑↓	↑↓	↓	↓	↓	↓	↑	↑
σ_y ↑	right	right	II, III	↑	↑↓	↑↓	↑	↑↓	↑↓	↑	↑	↑	↑	↓	↓
α ↓	left	none	I	↓	↑	↑↓	↑↓	↑↓	↓	↓	↑	↑	↓	↑	↓
λ ↓	left/right	none	I, III	↓↑	↑↓	↑↓	↓↑	↓	↑↓	↑	↓	↓	↑	↓	↑
A ↓	none	none	none	–	–	–	↑	↑↓	–	–	–	–	↑	↓	–

Notes: An increase in R&D subsidy rate $\sigma_R > 0$ or RPA tax rate $\sigma_X > 0$ yields the same qualitative effects as an increase in σ_y. The results for a decrease in λ are reported for the benchmark case of zero subsidies.

Appendix A: Derivation of Equilibrium Equations

The VC condition (27) is derived as follows. Start with substituting $r = \rho$ and $\dot{V}_y / V_y = 0$ in (16). Next, substituting $\alpha(1-\phi)Z = q(\theta)(V_a - V_y)$ from (10) and π_y from (9) into the resulting expression provides the following equation for V_y

$$V_y = \frac{Z\phi(\lambda - 1 + \sigma_y)}{\rho}. \qquad \text{(A.1)}$$

Combining (A.1) with (14) yields the following free-entry in R&D condition

$$w_H(1-\sigma_R)\beta\delta X = \frac{Z\phi(\lambda - 1 + \sigma_y)}{\rho} \qquad \textbf{FE}.\text{[46]} \qquad \text{(A.2)}$$

Substituting (21) into (17), using expressions $r = \rho$ and $\dot{V}_a / V_a = 0$ and taking limits as $dt \to 0$ yields

$$V_a = \frac{\pi_a - 2w_H \gamma X}{\rho}. \qquad \text{(A.3)}$$

Solve (A.2) for $w_H X$ and substitute the resulting expression and π_a from (6) into (A.3) to obtain

$$V_a = Z\left[\frac{\lambda - 1 + \sigma_a}{\rho} - \frac{2\phi(\lambda - 1 + \sigma_y)}{B(1-\sigma_R)\rho^2}\right], \qquad \text{(A.4)}$$

where $Z = cN/\lambda$ from (7) and choice of numéraire, i.e. $w_L \equiv 1$; and $B \equiv \beta\delta/\gamma$ is the resource requirement of R&D relative to RPAs. Substitute V_y from (A.1) and V_a from (A.4) into the vacancy-creation condition (10), and observe that Z cancels out. This yields the **VC** condition (27) in the main text.

The RP condition (28) is derived as follows. Substituting $r = \rho$, $\dot{V}_o / V_o = 0$ and $\pi_o = (1-\phi)cN(\lambda - 1)/\lambda$ into (20), and noting $Z = cN/\lambda$ yields

$$V_o = \frac{Z(1-\phi)(\lambda - 1)}{\rho + q}. \qquad \text{(A.5)}$$

Dividing (A.2) by (21) using V_y from (A.1), V_a from (A.4), and V_o from (A.5), yields the **RP** condition (28) in the main text.

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[46] Equations (A.1) and (A.2) illustrate the necessity of the assumption that an innovator captures a small fraction of the market immediately (i.e., $\phi > 0$). Where $\phi = 0$, the reward to R&D vanishes, i.e., $V_y = 0$ and there is no Schumpeterian growth and labor turnover.

Finally, the **creative-destruction** (**CD**) condition (29) is derived as follows. Setting $\dot{n}_A = 0$ in (5) yields $n_A = q(\theta)/[I + q(\theta)]$. Next, substitute this expression into (26) to obtain equation (29) in the main text.

Appendix B: Existence and Uniqueness of the Equilibrium

Solving (27) for q and simplifying implies that for $q > 0$, the following parametric restriction must hold

$$q = \frac{\alpha\rho}{\lambda - 1 + \frac{\sigma_a}{1-\phi} - \frac{\phi}{1-\phi}\left[\frac{2\left(\lambda - 1 + \sigma_y\right)}{\rho\left(1-\sigma_R\right)B} + \sigma_y\right]} > 0\,. \tag{B.1}$$

Because q is strictly declining in $\theta \equiv v/u$, condition (B.1) guarantees the existence and uniqueness of θ, and hence of $q(\theta)$ and $p(\theta)$. As a result to have unique $I > 0$, the denominator of (28) must be positive, which gives us our second parametric restriction

$$\frac{\lambda - 1 + \sigma_a}{\rho} - \frac{2\phi\left(\lambda - 1 + \sigma_y\right)}{B\left(1-\sigma_R\right)\rho^2} - \frac{(1-\phi)(\lambda - 1)}{\rho + q} > 0\,, \tag{B.2}$$

where q is given by (B.1). Conditions (B.1) and (B.2) must jointly hold for a unique interior equilibrium. Our numerical simulations show that these restrictions indeed hold for a wide range of empirically relevant parameters.[47] The existence of unique $u \in (0, 1)$ then follows from solving (29) for u yielding

$$u = \left[1 + \frac{p(I+q)}{qI(1-\phi)}\right]^{-1}.$$

Appendix C: Growth Rates

a. Growth Rate of Instantaneous Utility

We obtain the growth rate of instantaneous utility $h(t)$ as follows. Substituting $y(\omega,t) = c(t)/P(\omega,t)$ from (3) into (2); using $P_a(\omega,t) = P_y(\omega,t) = w_L/(\lambda^{m(\omega)-1})$ for a fraction $n_A + (1 - n_A)(1 - \phi)$ of industries; $P_o(\omega,t) = w_L/\lambda^{m(\omega)-2}$ for a fraction $(1 - n_A)\phi$ of industries, $w_L \equiv 1$; and taking into account that only a fraction n_A of industries are targeted for innovation at each point in time, provides

---

[47] The simulations use *Mathematica* version 8. The source programs are available upon request and also on the authors' websites.

$$
\begin{aligned}
\log h(t) &= \int\limits_{n_A} \log\left[\frac{c(t)}{P_a(t)}\right] d\omega + \int\limits_{(1-n_A)(1-\phi)} \log\left[\frac{c(t)}{P_o(t)}\right] d\omega + \int\limits_{(1-n_A)\phi} \log\left[\frac{c(t)}{P_y(t)}\right] d\omega \\
&= \int\limits_{n_A} \log\left[\frac{c(t)\lambda^{m(\omega,t)-1}}{w_L}\right] d\omega + \int\limits_{(1-n_A)(1-\phi)} \log\left[\frac{c(t)\lambda^{m(\omega,t)-2}}{w_L}\right] d\omega + \int\limits_{(1-n_A)\phi} \log\left[\frac{c(t)\lambda^{m(\omega,t)-1}}{w_L}\right] d\omega \\
&= n_A \log(c) + (1-n_A)(1-\phi)\log(c) + (1-n_A)\phi\log(c) \\
&\quad + \int\limits_{n_A} \log\left[\lambda^{m(\omega,t)-1}\right] d\omega + \int\limits_{(1-n_A)(1-\phi)} \log\left[\lambda^{m(\omega,t)-1}\lambda^{-1}\right] d\omega + \int\limits_{(1-n_A)\phi} \log\left[\lambda^{m(\omega,t)-1}\right] d\omega \\
&= \log(c) + \int_0^1 \log\lambda^{m(\omega,t)-1} d\omega + \int\limits_{(1-n_A)(1-\phi)} \log\lambda^{-1} d\omega \\
&= \log(c) - \log\lambda - (1-n_A)(1-\phi)\log\lambda + \int_0^1 \log\lambda^{m(\omega,t)} d\omega \\
&= \log(c/\lambda) - (1-n_A)(1-\phi)\log\lambda + (\log\lambda) n_A It.
\end{aligned}
$$

The last line, which is equation (30) in the main text, uses the property $\int_0^1 \log\lambda^{m(\omega,t)} d\omega$ = $(\log\lambda) I_{AGG} t$ of stochastic Poisson processes (see Grossman and Helpman 1991, p. 97). We note that $I_{AGG} = In_A$ captures the expected aggregate innovation rate. Every time a labor saving innovation takes place in a fraction n_A of the industries, a potential for a price decline by λ materializes and the value of the integral term $\int \log\lambda^{m(\omega,t)} d\omega$ increases by $\log\lambda$. This is the dynamic component of welfare due to technological progress.

The static component is the logarithm of the quantities consumed of goods summed over all industries, and it is given by $\log(c/\lambda) - (1 - n_A)(1 - \phi)\log\lambda$. The first term is standard, whereas the second term accounts for different prices charged by old and young firms: old firms charge a price $1/\lambda^{m(\omega)-2}$ in a share $1 - \phi$ of B industries, whose fraction is $1 - n_A$. This exceeds the price charged by young firms, $1/\lambda^{m(\omega)-1}$.

Differentiating expression $\log h(t)$ derived above with respect to time and taking into account $\dot{n}_A = \dot{c} =$ 0, which hold in the steady-state equilibrium, yields the growth rate of instantaneous utility

$$
\dot{h}/h \equiv g = In_A \log(\lambda) = Iq\log(\lambda)/(I+q).
$$

b. Growth Rate of Aggregate Output

The economy-wide output level at time t is calculated as follows. In each of the n_A growth-oriented industries, an adult firm produces $\lambda^m Z$ units of output. In the remaining $1 - n_A$ employment-oriented industries, a young firm produces $\phi\lambda^m Z$ units of output and an old firm produces $(1 - \phi)\lambda^{m-1} Z$ units of output. Hence aggregate output is given by

$$Y(t)=\int_{n_A} Z\lambda^{m(\omega,t)}d\omega+\int_{(1-n_A)(1-\phi)} Z\lambda^{m(\omega,t)-1}d\omega+\int_{(1-n_A)\phi} Z\lambda^{m(\omega,t)}d\omega .$$

The Poisson process governing the arrival of innovations implies that the expected number of innovations E[$m(\omega)$] in industry ω at time t equals $I(\omega)n_A t$. At each point in time only a fraction n_A of all industries are targeted for innovation; thus, $I(\omega)n_A$ is the expected innovation success rate in industry ω. The assumption of structurally identical industries implies that in the steady-state equilibrium we have E[$m(\omega)$] = m and thus:

$$Y(t)=n_A\lambda^m Z+\left(1-n_A\right)\lambda^m Z\left[\phi+(1-\phi)\lambda^{-1}\right]=\Psi\lambda^m Z ,$$

where $\Psi \equiv n_A + (1 - n_A)[\phi + (1 - \phi)\lambda^{-1}]$ is constant over time at the steady-state equilibrium. Taking logs and differentiating with respect to time, and using $E[m] = n_A It$, gives the following output growth rate:

$$\log Y(t)=\log\Psi+m\log\lambda+\log Z=\log\Psi+n_A It\log\lambda+\log Z \quad \Rightarrow \quad \dot{Y}/Y=n_A I\log\lambda .$$

c. Growth Rate of Prices

The growth rate of prices is derived as follows. We start with the price growth rate of a typical industry. In the present model, firm level prices remain the same during each phase and follow a stepwise process with each jump caused by a switch to firm status. At the industry level, however, process innovations generate downward price adjustments. Consider first an industry ω that is currently registered as an A industry, where $P^A = w_L/\lambda^{m(\omega)-1}$. Using E[$m(\omega)$] = $I(\omega)n_A t$ and $w_L \equiv 1$, one can calculate the growth rate of expected prices in this industry as

$$\log P^A(\omega)=\log(1)-\log\left[\lambda^{m(\omega)-1}\right]=-\left[m(\omega)-1\right]\log\lambda=-\left[I(\omega)n_A t-1\right]\log\lambda \quad \Rightarrow \quad \dot{P}^A/P^A=-I(\omega)n_A\log\lambda.$$

To determine the growth rate in expected goods prices in the ϕ and $1 - \phi$ segments of any industry currently registered as B industry, first note that $P^{B,\phi} = w_L/\lambda^{m(\omega)-1}$ and $P^{B,1-\phi} = w_L/\lambda^{m(\omega)-2}$, and then use derivations analogous to the case of an A industry analyzed above to obtain

$$\dot{P}^{B,\phi}/P^{B,\phi}=\dot{P}^{B,1-\phi}/P^{B,1-\phi}=-I(\omega)n_A\log\lambda .$$

Thus, we conclude that in any industry ω, regardless of which type of industry it is currently registered as, the rate of decline in expected price level is $I(\omega)n_A\log\lambda$. With structural symmetry across a continuum of industries, the growth rate in the aggregate price level is deterministic and given by $\dot{P}_{AGG}/P_{AGG}$ = $-n_A I\log\lambda$. It follows that per-capita consumption measured in units of output (i.e., c/P_{AGG}) grows at the rate $n_A I\log\lambda$.