A simple model of quality heterogeneity and international trade

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\textbf{ABSTRACT}

This paper develops a trade model with firm-specific quality heterogeneity in markets where firms face the threat of imitation and engage in limit-pricing strategies. Firms producing high-quality (high-price) products export, whereas firms producing lower-quality (lower-price) products serve the domestic market. Trade liberalization raises the average domestic markup and increases the number of products consumed in each country. However, the impact of trade liberalization on the average export markup depends on the nature of liberalization. Although the presence of markups renders the laissez-faire equilibrium suboptimal, trade liberalization increases national and global welfare.

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1. Introduction

Several empirical studies have documented the presence of substantial firm heterogeneity in narrowly defined product categories.\textsuperscript{1} According to these studies, firm heterogeneity takes the form of productivity or product-quality differences among establishments. More productive firms are larger, charge lower prices, and are more likely to engage in exporting. Firms producing higher-quality products charge higher prices and are more likely to engage in exporting.

The starting point of this paper is based on three observations. First, it is well known that firms in many R&D-intensive markets often face inelastic demand curves (especially during the early stages in the life of new products) coupled with threat of imitation. This situation arises in many R&D-intensive markets where the low sensitivity of demand can be traced to advertising, high switching costs, and high consumer-search costs. For example, Simon (1979, Table 4) reports that the demand for 17 out of 22 pharmaceutical brands analyzed is inelastic, with the average price elasticity during their life cycle period ranging from $-0.05$ to $-0.87$.\textsuperscript{2} Firms in these imperfectly competitive markets do not maximize profits by charging the unconstrained monopoly price, but usually resort to entry-preventing strategies to minimize the threat of...
imitation. Indeed, the combination of inelastic demand curves with threat of imitation has been the building blocks of Schumpeterian growth theory.

Second, several studies have documented the existence of significant fixed exporting costs (see, e.g., Roberts and Tybout, 1997). Third, a growing empirical literature has established that the effects of trade on firm profit margins measured by domestic and export markups are in general ambiguous. These observations raise several novel research questions. What are the effects of trade liberalization in markets where firms face inelastic demands and engage in Bertrand price competition with potential imitators? Does trade liberalization in these markets operate through the same channels as the ones proposed by the existing literature on firm heterogeneity and trade (Melitz, 2003; Melitz and Ottaviano, 2008)? What are the general-equilibrium effects of trade liberalization on domestic and export markups in the presence of significant fixed domestic costs and fixed exporting costs?

This paper develops a model of trade with firm-specific quality heterogeneity in which firms face potential imitators and positive fixed costs when entering the domestic and exporting markets. Following the insights of Schumpeterian growth theory, we introduce Bertrand price competition between a firm that discovers a new variety and a competitive fringe of imitators that results in entry-deterring limit prices. We also assume that each firm faces a unitary-elastic demand curve, which is derived from a Cobb–Douglas utility function.

In the model, firms producing higher-quality products charge higher prices and markups, enjoy higher profits, and export their products. In contrast, firms producing lower-quality products charge lower prices, earn lower profits, and serve only the domestic market. Trade liberalization increases the number of varieties available for consumption; reallocates resources from low-quality to high-quality products that generates exit of inefficient firms; and improves national and global welfare. Trade liberalization increases the average domestic markup. However, the effects of trade liberalization on the average export markup depend on the nature of that liberalization: a move from autarky to restricted trade or an increase in the number of trading countries increases the average export markup; a decline in foreign-market entry costs reduces the average export markup; and a reduction in per-unit trade costs has an ambiguous effect on the average export markup. We also present a modified version of our model where firms face spatial marginal costs of production. In the modified version of our model, firms with higher-quality products charge higher prices in more distant markets; and enjoy higher markups, larger revenues and larger market shares. More firms export to more proximate markets, and sufficiently high foreign-market-entry costs eliminate bilateral trade flows between distant markets. Finally, we find that the laissez-faire equilibrium is inefficient. This inefficiency can be traced to the difference between the socially optimal and market-based average markups.

Our model offers several predictions that are consistent with empirical findings. For instance, the prediction that each surviving firm charges a price that is positively correlated to its product quality level is consistent with empirical studies that routinely use unit values to measure product quality (Schott, 2004; Hummels and Skiba, 2004; Hallak, 2006). Furthermore, the prediction that the average quality (and price) of exports is higher than the average quality (and price) of products sold only in the domestic market provides a novel general-equilibrium explanation based on quality sorting of Alchian and Allen (1964, pp. 74–75) conjecture of shipping the good apples out. This prediction is also supported by several empirical studies (Hummels and Skiba, 2004; Baldwin and Harrigan, 2011; Verhoogen, 2008). The model’s prediction that productivity (and in our case quality) differences account for higher export markups is consistent with the findings of De Loecker and Warzynski (forthcoming).

Our paper is related to a growing theoretical literature on firm heterogeneity, trade and markups. Bernard et al. (2003) (BEJK, henceforth) develop a model in which firm heterogeneity arises from firm-specific Ricardian comparative advantage. Their model assumes that firms engage in limit-pricing strategies resulting in an endogenous distribution of markups. However, their model assumes that the number of varieties produced/consumed in the world is exogenously fixed.

Our model is more closely related to Melitz (2003) and Melitz and Ottaviano (2008) who address formally the effects of trade liberalization in markets characterized by productivity-based firm heterogeneity. Melitz (2003) focuses on the effects of trade liberalization on aggregate productivity and welfare in markets where firms face positive production and exporting costs and charge the unconstrained monopoly price. Melitz and Ottaviano (2008) focus on the effects of trade on markups, aggregate productivity, and welfare in the case where firms face zero fixed production and fixed exporting costs and charge the unconstrained monopoly price.

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1. Levinsohn (1993) argued that import competition reduced the average domestic markups in Turkish manufacturing firms. Chen et al. (2009) find that trade openness reduces average domestic markups in the short-run but has no long run effects. Using plant-level data from Cote d’Ivoire, Harrison (1994) analyzed changes in markups following a 1985 trade liberalization episode which reduced trade costs. She found that markups increased in five out of nine sectors and declined in others following the reforms.

2. Segerstrom et al. (1990) and Grossman and Helpman (1991), among many others, provide more details on this type of market structure.

3. Under Dixit and Stiglitz (1977) preferences and the threat of imitation, each firm faces a kinked demand curve. In this case, the profit-maximizing entry deterring strategy can be either a constant monopoly markup or a limit price depending on the quality level.

4. Feenstra (2004) reports that the average price and quality of Japanese exported cars to the US was higher than the corresponding average price and quality of domestic US cars. For instance, according to Table 8.3 on page 274, in 1979 (two years before the auto VER was imposed on Japanese cars) the unit value of Japanese cars imported into the US was $4949 compared to $4186 of small US cars. The corresponding unit-quality values for imported Japanese and domestic US cars were $4361 and $4197, respectively.

5. de Blas and Russ (2010) generalize the BEJK model by introducing a discrete number of potential entrants (team of rivals) competing in a Bertrand fashion with each incumbent firm. Their model delivers an endogenous distribution of markups but inherits several features from the BEJK model. These features include an exogenously given number of varieties, absence of foreign-market entry fixed costs, and a specific distribution of productivity levels among firms.
This paper differs from theirs in several ways. First, firms in our model charge constrained monopoly prices and compete against potential imitators. Second, unlike theirs, our model allows us to investigate the effects of changes in fixed exporting costs on average domestic and export markups. Finally, in our model trade affects each economy mainly through income-based shifts in residual demand. In Melitz (2003) trade liberalization works through changes in the real wage, and in Melitz and Ottaviano (2008) it works through changes in the price elasticity of demand. Despite these differences, all three models predict that trade raises national and global welfare.

Baldwin and Harrigan (2011) and Kugler and Verhoogen (2012) introduce a quality dimension in Melitz (2003) model. Like ours, these models predict that more productive firms charge higher prices; but unlike our model, they deliver constant markups. Moreover, in their models, firms cannot be partitioned by export status unless there are a sufficiently high fixed foreign-market entry costs. In contrast, our model even delivers the partition of firms by export status with no fixed entry costs associated with exporting.

In summary, this paper makes several novel contributions to the theory of firm heterogeneity and trade. First, it offers a unique and tractable analytical framework of quality-based firm heterogeneity where firms face unitary elastic demand curves, the threat of imitation, and engage in entry preventing limit-pricing strategies. The model delivers heterogeneous prices and markups and allows the study of trade liberalization that is transmitted primarily through an income-based mechanism that changes the intensity of product market competition. This general-equilibrium mechanism is missing from the rest of the literature. Second, the proposed framework offers several predictions on the pattern of prices, market shares, and average markups in markets populated by vertically differentiated products and characterized by significant foreign-market-entry costs that are missing from the literature. Third, our model demonstrates the sub-optimality of the laissez-faire equilibrium and opens the possibility for studying the nature of welfare improving policies.

The rest of the paper is organized as follows. Section 2 presents the basic elements of the model and the steady-state equilibrium. Section 3 analyzes the impact of trade liberalization and the effects of a move from autarky to restricted trade. Section 4 augments the original model by adding spatial cost considerations. Section 5 describes the model’s welfare properties. Section 6 concludes.

2. The model

In this section, we present the basic elements of the model regarding consumer preferences, structure of production, and firm entry decisions. We consider a global economy consisting of \( n + 1 \) structurally identical countries with \( n \geq 1 \). Each economy has a single industry populated by heterogeneous firms, and labor is the only factor of production. In each country, the aggregate supply of labor, \( L \), is fixed and remains constant over time.

### 2.1. Consumer preferences

Consumer preferences are identical across all countries and modeled by the following Cobb–Douglas utility function defined over a continuum of products indexed by \( \omega \)

\[
U = \int_{\omega \in \Omega} \ln \left( \beta \lambda(\omega) \frac{q(\omega)}{L} \right) d\omega,
\]

where \( \beta > 0 \) is a constant, \( \lambda(\omega) \) denotes the time-invariant product quality, \( q(\omega) \) is the aggregate consumption of brand \( \omega \), and \( \Omega \) is the set of varieties available for consumption in a typical country. We focus our analysis on the case where each consumer buys all available varieties, that is, we assume that the love-for-varieties principle holds. This case arises if parameter \( \beta \) is sufficiently high to ensure that the utility increases monotonically in the mass of varieties consumed.\(^8\)

Maximizing (1) subject to the budget constraint yields the standard Cobb–Douglas demand for a typical variety

\[
q(\omega) = \frac{EL}{p(\omega)M_t},
\]

where \( E \) is per-capita consumer expenditure, \( L \) is the number of consumers in a typical (home or foreign) market, \( p(\omega) \) is the corresponding price of brand \( \omega \), and \( M_t \) is the measure of \( \Omega \) (i.e., the mass of varieties available for consumption). The market demand for a product increases in aggregate consumer expenditure \( EL \); and decreases in price \( p(\omega) \) and the number of available products \( M_t \).

### 2.2. Production

There is a continuum of firms, each choosing to produce a different product variety. Labor is the only factor of production, with each worker supplying one unit of labor, and wage rate is normalized to one. Production of a brand with

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\(^8\) The condition \( \beta > eL/f_c \), where \( e \) is the natural logarithm base and \( f_c \) is the fixed foreign market entry cost, guarantees the “love-for-variety” property inherent in Dixit and Stiglitz (1977) preferences. Section 2.4 provides more details on its derivation. We would like to thank Tetsu Haruyama for pointing this out.
quality $\lambda$ involves both fixed and variable costs: in order to produce $q$ units of output, $\ell = f_d + \lambda q$ units of labor are required, where $f_d$ denotes the fixed overhead cost of production measured in units of labor, and $\theta > 0$ is a parameter.9

Firms wishing to export must incur per-unit trade costs and fixed costs. Iceberg trade costs (such as transport costs and tariffs) are modeled in the standard fashion: $\tau > 1$ units of output must be produced at home in order for one unit to arrive at its destination. In addition, exporting involves a fixed foreign-market-entry cost of $F_x > 0$ that does not depend on the firm’s quality level or the geographic location of production.10 The decision to export occurs after the product’s quality is revealed.

Each incumbent firm faces a constant probability of death $\delta$ in each period. In the present context, this stochastic shock can be interpreted as adverse changes in tastes that eliminate the demand for a particular variety. Consequently, in the steady-state, each firm is indifferent in principle between paying $f_d = \delta F_x$ in each period and the one-time fixed cost $F_x$ in the first period of its existence. Hereafter, we assume that in each period exporters face an overhead fixed cost $f_d$ in addition to the overhead production cost $f_d$. Firms that serve only the domestic market face just the overhead production cost $f_d$.

Next, consider the optimal pricing decision of a firm selling a brand of quality $\lambda$ in its home market. Because each brand is associated with a unique quality level, in what follows we label products based on their quality levels. The brand’s quality level, in what follows we label products based on their quality levels. The quality $\lambda$ involves both fixed and variable costs: in order to produce $q$ units of output, $\ell = f_d + \lambda q$ units of labor are required, where $f_d$ denotes the fixed overhead cost of production measured in units of labor, and $\theta > 0$ is a parameter.9

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Next, consider the optimal pricing decision of a firm selling a brand of quality $\lambda$ in its home market. Because each brand is associated with a unique quality level, in what follows we label products based on their quality levels. The aggregate quantity demanded is given by Eq. (2), which implies that the expenditure per variety, $p(\lambda)q(\lambda)$, is independent of the brand’s quality level. Because the elasticity of demand for each variety is unity, a typical firm has an incentive to charge an infinite price and produce an infinitesimally small quantity independently of the product’s quality level. To prevent this from happening and to create an endogenous distribution of markups, we follow the spirit of Schumpeterian growth theory and assume that once a product is introduced in a market (domestic or foreign), a generic, lower-quality version of the product can be produced instantaneously by a competitive fringe of firms. We assume that the successful imitator of a product with quality $\lambda$ can produce a generic product with quality $\lambda_0$. Each imitator faces the same marginal costs of production as a successful innovator: in order to produce $q$ units of output an imitator must hire $\ell = \lambda_0 q$ units, where $\lambda$ is the quality of the original (as opposed to the generic) product. We suppose that the imitated version of a product cannot be produced in a country unless the original product is sold there. In other words, the technology to produce generic products diffuses internationally through imports.11 We normalize the quality level of each generic to one independently of the quality level of the copied product and the location of production (i.e., $\lambda_0 = 1$).

Denote with $p_d(\lambda)$ and $p_q(\lambda)$ the consumer price prevailing in the domestic and foreign markets respectively, and assume that competition within each product occurs in a Bertrand fashion. The possibility of costless imitation forces firms to maximize profits by charging a (limit) price not higher than $p_d(\lambda) = p_q(\lambda) = w\lambda^{1+\theta}$, where $w$ is the common wage rate across all countries, normalized to unity. This optimal pricing rule drives domestic and foreign imitators out of the market and implies that firms with higher-quality products charge higher prices.12

Consider then two firms producing products with quality levels $\lambda_1$ and $\lambda_2$. The limit-pricing rule $p_d(\lambda) = p_q(\lambda) = \lambda^{1+\theta}$ and (2) yield

\begin{equation}
\frac{q(\lambda_2)}{q(\lambda_1)} = \frac{p_d(\lambda_1)}{p_d(\lambda_2)} = \left(\frac{\lambda_1}{\lambda_2}\right)^{1+\theta},
\end{equation}

which means that firms with higher-quality products charge higher prices and sell lower quantities.

The per-period profits of exporting firms can be decomposed into two parts: profits earned from domestic sales $\pi_d(\lambda)$, and profits earned from sales in each of $n$ export markets $\pi_e(\lambda)$.

\begin{align}
\pi_d(\lambda) &= [p_d(\lambda) - \lambda^0]q_d(\lambda) - f_d = (1-\lambda^{-1})\frac{E\ell}{M_c} - f_d, \\
\pi_e(\lambda) &= [p_q(\lambda) - \tau\lambda^0]q_e(\lambda) - f_x = (1-\tau\lambda^{-1})\frac{E\ell}{M_c} - f_x.
\end{align}

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9 Since the quality parameter $\lambda$ is time-invariant and exogenous, no further restrictions are needed for the parameter $\theta$. However, if firms had an option to upgrade the quality of their products, then profit-maximization considerations would require $\theta > 1$ to ensure convexity of the cost function with respect to $\lambda$. Incorporating endogenous quality choice in the model constitutes a useful extension which is left for future research.

10 Existence of such market costs of exporting have been well documented by several studies (e.g., Roberts and Tybout, 1997; Bernard and Jensen, 1995).

11 Alternatively, one can assume that once a product is developed, its low-quality generic version can be produced by a competitive fringe in all countries, i.e., technology diffuses instantly across all countries. Analysis based on this assumption mainly yields qualitatively the same results, and is available upon request. Moreover, one can also assume that there is no international transfer of technology. This assumption would allow exporters to charge a higher price abroad that would be proportional to the product’s quality level adjusted by per-unit trade costs. More precisely, in the absence of international technology transfer, the quality leader charges two limit prices: $p_d(\lambda) = \lambda^{1+\theta}$ in the domestic market to get rid of the competitive fringe; and (in the absence of a competitive foreign fringe) it charges an export limit price $p_q(\lambda) = \lambda^{1+\theta}$ which prevents the domestic fringe from exporting. In this case, a reduction in trade costs lowers the average export markup. The rest of the main results hold in this case as well.

12 Appendix shows formally how an augmented version of the consumer utility function (1) can generate the aforementioned optimal limit-pricing rule.
Because only a fraction of incumbent firms export, a firm producing a good with quality $\lambda$ earns a per-period profit $\pi(\lambda) = \pi_d(\lambda) + \max(0, n\pi_0(\lambda))$. Since each firm faces a constant probability of death $\delta$ in each period, the market value of a typical firm is then given by $w(\lambda) = \max(0, \pi(\lambda)/\delta)$.

According to Eqs. (4a) and (4b), the flow of profits in each market is an increasing and monotonic function of quality $\lambda$. Because $\pi_d(1) = -f_d < 0$ and $\pi_x(0) = -f_x < 0$, there exist two cutoff quality levels $\lambda_d$ and $\lambda_x$ such that $\pi_d(\lambda_d) = \pi_x(\lambda_x) = 0$. Thus,

$$\lambda_d = \frac{1}{1-Mf_d/EL} > 1, \quad \lambda_x = \frac{\tau}{1-Mf_d/EL} \geq \tau > 1,$$

which further imply that

$$\lambda_x = \frac{\tau}{1-(1-\lambda_d^{-1})f_x/f_d}.$$

The requirement that the denominator of (6) must be non-negative implies that $\lambda_d \in (1,k)$, where $k = 1/[1-f_d/f_x]$. Inspection of (6) also indicates that $\lambda_x$ is a monotonically increasing function of the domestic cutoff quality level $\lambda_d \in (1,k)$, the level of per-unit trade costs $t$, and the ratio of overhead fixed costs $f_x/f_d$. In addition, if $f_x = f_d$, then (6) implies that $\lambda_x = \tau \lambda_d > \lambda_d$ for all $\lambda_d \in (1,\infty)$. Therefore, under the parameter restrictions $f_x \geq f_d$ and $\tau > 1$, the exporting cutoff quality level $\lambda_x$ is strictly greater than the domestic cutoff quality level $\lambda_d$. In this case, firms whose product quality level is less than $\lambda_d$ exit; firms whose product quality level is $\lambda \in [\lambda_d, \lambda_x)$ produce exclusively for the domestic market because they earn non-negative profits from the domestic operations only; and firms producing high-quality products ($\lambda \geq \lambda_x$) sell their products in both domestic and all foreign markets.

Although the condition $f_x \geq f_d$ is sufficient for the partition of firms by their export status, it is not necessary. Eq. (6) implies that $\lambda_x > \lambda_d$ if and only if $f_x/f_d > 1 - (\tau - 1)/(\lambda_d - 1)$. Two remarks are in order. First, note that the last inequality will always hold as long as $\tau > \lambda_d$. In this case, there is no restriction to impose on $f_x$. Second, as shall be shown in Lemma 2, $\lambda_d$ is inversely related to $\tau$. This ensures that the fixed cost $f_x$ can be smaller than $f_d$ as long as the transport cost $\tau$ is sufficiently high. Thus, the results of this paper still hold under the alternative assumption that the combination of fixed and variable trade costs are high enough to endogenously divide firms into exporters and non-exporters. A similar condition can be derived for the case of Dixit and Stiglitz (1977) preferences.\footnote{When preferences are in the Dixit and Stiglitz form as in the Melitz model, the necessary condition for the partition of firms is $\tau = 1/f_d > f_d$ (see Melitz, 2003, p. 1709). Since $\tau > 1$, the restriction that $f_x > f_d$ is also a sufficient condition in the Melitz model. Moreover, as in our model, $f_x$ can be smaller than $f_d$ as long as the variable trade cost $\tau$ is sufficiently high.}

**Proposition 1.** Let $\lambda_d$ and $\lambda_x$ denote the domestic and export cutoff quality levels, respectively; and let $k = 1/[1-f_d/f_x]$ denote the upper bound of the domestic cutoff quality level $\lambda_d$. Then,

a. The export cutoff quality level $\lambda_x$ is an increasing function of the domestic cutoff quality $\lambda_d$.

b. If the production fixed cost does not exceed the foreign-market entry cost (i.e., $f_d \leq f_x$), then the export cutoff quality level is strictly greater than the domestic cutoff quality level (i.e., $\lambda_x > \lambda_d > 1$).

c. In the absence of foreign market entry cost (i.e., $f_x = 0$), there exists a level of trade cost such that the export cutoff quality level is strictly greater than the domestic cutoff quality level (i.e., $\lambda_x = \tau > \lambda_d > 1$).

**Proposition 1** implies that firms with high-quality products charge higher prices, enjoy higher profits and ship these products abroad. As such, it offers a general-equilibrium explanation for Alchian and Allen (1964) conjecture which states that a per-unit transaction cost lowers the relative price of, and increases the relative demand for high-quality goods. The hypothesis that high-quality goods are exported has been confirmed empirically by the work of Hummels and Skiba (2004). **Proposition 1** highlights how supply-based factors and general-equilibrium interactions, such as per-unit trade costs and self-selection into exporting among heterogeneous firms, can lead to shipping high-quality products abroad.

2.3. Entry decision

We assume that there is a large number of ex-ante identical entrants, each faces a fixed entry cost $f_e > 0$, which is measured in units of labor. After a firm incurs the fixed entry cost, it draws its quality parameter $\lambda$ from a common and known distribution $g(\lambda)$ with positive support over $(0,\infty)$ and with continuous cumulative distribution $G(\lambda)$. The ex-ante probability of drawing a quality level $\lambda$ is governed by the density function $g(\lambda)$ and the ex-ante probability of successful entry $1 - G(\lambda_d)$. 
Thus, the ex-post distribution of product quality levels $\psi$ is the conditional distribution of $g(\lambda)$ on the interval $[\lambda_d, \infty)$

$$\psi(\lambda) = \begin{cases} \frac{g(\lambda)}{1-G(\lambda_d)} & \text{if } \lambda > \lambda_d \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

The ex-ante probability that an incumbent firm will export to each market is given by

$$\xi_a = \frac{1-G(\lambda_a)}{1-G(\lambda_d)} \tag{8}$$

In addition, the law of large numbers implies that $\xi_a$ equals the ex-post fraction of incumbent firms that export. Thus, if $M_d$ is the mass of varieties (and firms) produced in any country, then $M_a = \xi_a M_d$ is the number of varieties that each country exports. This further ensures that $M_a = (1 + n_{c1}) M_d$ is the mass of products available for consumption in any country.

Armed with the aforementioned probability distributions, using Eqs. (4a) and (4b), one can easily calculate the average per-period profit of a successful entrant

$$\pi = (1 - \lambda_d^{-1}) \frac{EL}{Ec} dH - \frac{\varphi}{dH} \left[ (1 - \lambda_d^{-1}) \frac{EL}{Ec} g(\lambda) \right] \tag{9}$$

where $\lambda_i (i = d, x)$ is given by

$$\lambda_i = \frac{1}{1-G(\lambda_i)} \int_{\lambda_i}^{\infty} \lambda^{-1} f(\lambda) \, d\lambda \tag{10}$$

That is, $\lambda_i$ is the weighted harmonic mean of the quality levels (and prices) of all produced goods and can be interpreted as the average (expected) quality level. Similarly, $\lambda_x$ is the weighted harmonic mean of the quality levels of a country's exports and can be interpreted as the average export quality.

Since the domestic cutoff quality level is the minimum quality level of all surviving products, it must be lower than the average quality level. Moreover, an increase in $\lambda_d$ forces producers with low-quality products to exit the market, which in turn increases the average quality level of all produced varieties. The same intuition applies to the relationship between the export quality cutoff level and the average quality of exports.

**Lemma 1.** The average quality level of all products produced in a typical market is strictly greater and increases in the domestic cutoff quality level, i.e., $\lambda_d > \lambda_d$ and $\frac{\partial \lambda_d}{\partial \lambda_d} > 0$. The average quality level of exports is strictly greater and increases in the export cutoff quality level, i.e., $\lambda_x > \lambda_x$ and $\frac{\partial \lambda_x}{\partial \lambda_x} > 0$.

Because the probability of successful entry is $1 - G(\lambda_d)$, the net benefits of entering the domestic market are equal to the expected value of a firm $[1-G(\lambda_d)]\pi$, where $\pi = \varphi/\delta$ is the average value of a successful entrant

$$[1-G(\lambda_d)]\frac{\pi}{\delta} = f_d \tag{11}$$

where $\pi$ is defined by (9).

2.4. Steady-state equilibrium

Using (6) and (8) in (9) yields an expression for ex-ante profits that depends only on the two cutoff quality levels. Further substitution of $\pi$ into the free-entry condition (11) yields the basic steady-state equilibrium condition

$$f_d H(\lambda_d, 1) + n f_x H(\lambda_x, \tau) = \delta f_e \tag{12}$$

where $H$ is defined as

$$H(\lambda_i, x) = [1-G(\lambda_i)] \left[ \frac{1-\lambda_i^{-1}}{1-2\lambda_i^{-1}} \right] \tag{13}$$

where again $u_i = 1, \tau$ for $i = d, x$, respectively.

The export cutoff quality level $\lambda_x$ is an increasing function of $\lambda_d$ (see Proposition 1). In addition, Appendix proves that $H$ is strictly decreasing in cutoff levels and that the left-hand-side of Eq. (12) is also decreasing in $\lambda_d \in (1, k)$. The first term in the left-hand-side of (12) equals the current (as opposed the present) value of expected domestic profits expressed as a function of the domestic cutoff quality level $\lambda_x$. An increase in the latter affects expected domestic profits through three distinct channels. First, a higher quality level increases the flow of profits of a surviving firm by allowing this firm to charge a higher price. Second, a higher cutoff quality level decreases the probability of success and thus reduces the expected flow of profits. And third, a higher cutoff quality level increases the profit flow of the marginal producer and requires a lower expenditure level to bring the

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14 Substitute $\lambda_d^{-1} > \lambda_x^{-1}$ in the integral expression of Eq. (10) to obtain $\lambda_x^{-1} = \frac{\int \lambda_x^{-1} g(\lambda) \, d\lambda}{(1-G(\lambda_d)) - \lambda_d^{-1}}$, which yields $\lambda_x > \lambda_d$. Differentiation of (10) yields $\frac{\partial \lambda_d}{\partial \lambda_d} = \frac{\partial \lambda_x}{\partial \lambda_d} (\lambda_x^{-1} - \lambda_d^{-1}) > 0$. The same logic and calculations apply to $\lambda_x$ defined in (9).
flow of profits earned by the marginal firm down to zero. Appendix shows that the latter two effects dominate and therefore the current expected flow of domestic profits is decreasing in the domestic cutoff quality level. Similar considerations apply to the second term of the left-hand-side of (12) which equals the current expected flow of profits from exporting. Thus, we have the following result (see Appendix for proof).

**Proposition 2.** Let \( k = 1/(1-f_d/f_e) \) and assume that \( f_x \geq f_d > 0 \). There exist a unique domestic and a unique export cutoff quality levels \( \zeta_d \in (1,k) \) and \( \zeta_x \in (\tau, \infty) \) which satisfy Eqs. (6) and (12) such that \( \zeta_d > 1, \zeta_x > \tau, \) and \( \zeta_x > \zeta_d \).

Once the two cutoff quality levels are determined, one can solve for the values of the remaining endogenous variables. We start with the determination of the mass of products produced in each market. In the steady-state equilibrium, the per-period flow of successful entrants must be equal to flow of incumbents who exit the market because they are hit by a bad shock, i.e., \([1-G(\zeta_d)]M_e = \partial M_d/\partial \zeta_d\), where \( M_e \) is the mass of all (as opposed to successful) entrants. Then the aggregate amount of labor employed by prospective entrants is \( L_e = M_d f_e = \partial M_d f_e/[1-G(\zeta_d)] = M_d \pi \), where the last equality follows from the free-entry condition (10). Thus the aggregate amount of labor devoted to R&D equals the level of aggregate profits earned by all producers in a typical market.

The aggregate demand for labor in a typical market equals the aggregate supply of labor \( L_p + L_e = L_p + II = L \), where \( L_p \) denotes the total amount of labor employed in the production of surviving goods and \( II \) is the level of aggregate profits earned by all producers. In addition, the standard GDP identity implies that the total wage bill must be equal to the aggregate expenditure on all goods produced \( wL_p + wL_e = EL \). Therefore, per-capita expenditure equals unity due to the choice of labor as the numeraire, i.e., \( E = w = 1 \).

Substituting \( E=1 \) into (4a) and using \( \pi_d(\zeta_d) = 0 \) yields the mass of products available for consumption

\[
M_e = \left( 1 - \frac{\zeta_d^{-1}}{1 + n_{c_d}} \right) \frac{L}{f_d}. \tag{14}
\]

Observe that the mass of products available for consumption increases in the domestic cutoff quality level \( \zeta_d \). The intuition behind this result is straightforward and based on the zero-profit condition \( \pi_d(\zeta_d) = 0 \). An increase in the domestic cutoff quality level increases the flow of profits of the marginal producer and requires a reduction in demand for the marginal variety. The reduction in demand is achieved through a higher variety of products (higher intensity of product-market competition) that delivers a lower expenditure per variety \( L/f_dM_e \). Substituting the relationship \( M_e = (1 + n_{c_d})M_d \) into (14) yields the mass of varieties produced in each country

\[
M_d = \frac{1 - \zeta_d^{-1}}{1 + n_{c_d}} \frac{L}{f_d}. \tag{15}
\]

As mentioned in the introduction, the model generates an endogenous distribution of markups. Each incumbent firm charges a price \( p_d(\zeta) = \lambda^{1+\theta} \), and incurs a marginal cost \( c_d = \lambda^\theta \). As a result, its markup, measured by the price marginal-cost ratio, is given by \( p_d/c_d = \lambda \). Subsequently, the aggregate markup over all incumbents equals \( \int_{\zeta_d}^{\infty} \lambda M_d \psi(\lambda) d\lambda = M_d \bar{\lambda}_d \), where \( \bar{\lambda}_d = \int_{\zeta_d}^{\infty} \lambda \psi(\lambda) d\lambda \) denotes the average quality of domestically produced varieties. Similarly, an exporter charges a price \( p_e(\zeta) = \lambda^{1+\theta} \) and incurs a marginal cost \( c_e = \lambda^\theta \). Its markup is given by \( p_e/c_e = \lambda/\tau \). Thus the aggregate export markup is \( \int_{\zeta_e}^{\infty} (\lambda/\tau) M_d \psi(\lambda) d\lambda = M_d \bar{x}_e/\tau \), where \( \bar{x}_e = \int_{\zeta_e}^{\infty} \lambda \psi(\lambda) d\lambda \) denotes the average quality of exported varieties.

\[
\mu_d = \bar{\lambda}_d = \int_{\zeta_d}^{\infty} \lambda \psi(\lambda) d\lambda \quad \text{and} \quad \mu_x = \bar{x}_e/\tau = \frac{1}{\tau} \int_{\zeta_e}^{\infty} \lambda \psi(\lambda) d\lambda, \tag{16}
\]

where \( \psi(\lambda) \) is the ex-post distribution of domestic and export quality levels defined by (7).16

Finally, substituting the per-capita quantity demanded \( q(\lambda)/L = E/\lambda M_e \) for each variety into the utility function (1) and performing the integration yields

\[
U = M_e \ln \left( \frac{\beta E}{M_e} \right) = M_e \ln \left( \frac{\beta}{M_e} \right), \tag{17}
\]

where per-capita expenditure \( E \) is set equal to unity due to the choice of labor as the numeraire. Since \( \ln(\beta/M_e) > 1 \) (see footnote 16), \( U \) is always positive. Observe that per-capita welfare increases monotonically in the mass of varieties

---

15 We can now derive a sufficient condition which guarantees that the consumer consumes all available varieties. In principle, each consumer chooses \( q(x) \) (the quantity of each variety) and \( M \) (the number of varieties) to maximize (1) subject to the budget constraint. The first order condition with respect to \( M \) yields \( \ln(\beta M_e)/(\partial M_e/\partial M) \geq 1 \), which holds with equality if \( M = M_e \). Inserting (2) into this condition yields \( \ln(\beta M_e)/(\partial M_e/\partial M) \geq 1 \). Thus to ensure that the principle of non-satiation holds (i.e., \( M = M_e \)), we must have that \( \ln(\beta M_e)/(\partial M_e/\partial M) \geq 1 \). Since \( \partial M_e/\partial M = \lambda M_e \) and \( E = 1 \), we must have \( \ln(\beta M_e) > 1 \), that is \( \beta > eM_e \), where \( e \) is the base of the natural logarithm. Using (14) implies that \( \beta f_d/EL > 1 - \lambda^{-1} \). Moreover, because \( \zeta_d \in (1,k) \), the inequality condition holds if \( \beta f_d/EL > 1 - \lambda^{-1} \), which in turn yields \( \beta > f_e f_d \).

16 Markups can also be measured by the price (marginal) cost margin. In this case, the domestic and export markups are given by \( (p_d-c_d)/c_d = 1 - \lambda^{-1} \) and \( (p_e-c_e)/c_e = 1 - \lambda^{-1} \), respectively. As a result, the average domestic and export markups are \( \rho c_d = 1 - \lambda^{-1} \) and \( \rho c_e = 1 - \lambda^{-1} \), where \( \lambda \) and \( \lambda_e \) are defined by (10). The analysis based on these markup definitions yields the same qualitative results.
consumed $M_c$ and is independent of the average quality level. The latter depends positively on the expenditure per variety $E/M_c$, and on the domestic cutoff quality level $\lambda_d$ as indicated by (14).

The absence of average quality from the welfare expression (17) deserves a few remarks. First, the assumption of Bertrand price competition that leads to limit pricing and the assumption of perfect substitutability between different quality products within each variety imply the following: each consumer is indifferent between consuming a low-quality, low-price good that could be produced by a fringe of potential imitators and a high-quality, high-price good that is produced by an incumbent. In other words, per-capita utility for each variety consumed is $\lambda q(l)/L = E/M_c$ and therefore depends only on expenditure per variety. Second, the assumption of Cobb–Douglas preferences implies that consumer expenditure is distributed equally across all consumed varieties independently of their price (i.e., quality). Third, higher-quality allows incumbents to charge higher prices, produce lower quantities and save in manufacturing labor. As a result, any given endowment of labor can produce more higher-quality products and lead to higher per capital welfare. In other words, quality affects welfare indirectly through its impact on the mass of produced varieties. Fourth, observe that per-capita welfare is a concave function of varieties consumed $M_c$ implying diminishing returns to consumption of more varieties. The gains from trade due to importation of more varieties diminish because marginal varieties have lower quality. Arkolakis et al. (2008) highlight this “curvature” property for the case of CES preferences and document its magnitude using data from Costa Rica.

3. The impact of international trade

The model is well suited to analyze the general equilibrium effects of trade liberalization measured by an increase in the number of trading partners $n$, a reduction in per-unit trade costs $\tau$, and a reduction in foreign-market entry costs $f_x$. The impact of trade liberalization is channeled through two interacting general-equilibrium channels: changes in the demand for labor, which are captured by changes in the real wage; and changes in the intensity of product-market competition, which are captured by changes in the average markup and the mass of varieties available in each market. Formally, the effects of trade liberalization are transmitted through changes in the domestic cutoff quality level $\lambda_d$ as are described in Lemma 2 (see Appendix for proof).

**Lemma 2.** Suppose that the level of fixed foreign-market-entry costs is equal to or greater than the level of fixed overhead production costs (i.e., $f_x \geq f_d > 0$). Trade liberalization, captured by an increase in the number of trading partners $(n \uparrow)$, a reduction in per-unit transport costs $(\tau \downarrow)$, or a reduction in foreign-market entry costs $(f_x \downarrow)$, raises the domestic cutoff quality level $\lambda_d$ (i.e., $d\lambda_d/dn > 0$, $d\lambda_d/d\tau < 0$, and $d\lambda_d/df_x < 0$).

The economic intuition behind Lemma 2 is as follows. For any initial value of the domestic cutoff quality level $\lambda_d$, the export cutoff quality level $\lambda_x$ and the mass of varieties consumed $M_c$ are fixed (see Eqs. (6) and (14)). Eq. (6) and Lemma 1 imply that a decline in $\tau$ or $f_x$ increases the average quality of exports $\lambda_x$. Therefore, any form of trade liberalization $(n \uparrow$, $\tau \downarrow$, or $f_x \downarrow$) increases average profits $\Pi$ (see Eq. (9)) and raises the demand for labor for any wage level (see Eq. (11) and, in particular, Eq. (12)). The excess demand for labor induces a reallocation of resources from low-quality products towards high-quality products that translates into a larger mass of products available for consumption $M_c$. To see this, recall that for any level of expenditure, a firm with a higher quality product charges a higher price, produces less output, and employs less labor than a firm with a lower quality product (see Eq. (3)). Thus any given aggregate supply of labor can sustain more higher-quality products.

The reallocation of resources from lower to higher-quality products caused by the availability of more varieties intensifies the product–market competition by reducing the demand for each variety as the aggregate consumer income $E$ is spread among more varieties. Consequently, the zero-profit condition, which defines the domestic cutoff quality level, becomes negative when evaluated at the initial equilibrium, and requires a higher domestic cutoff quality level to restore profits back to zero. How does trade liberalization affect the export cutoff level $\lambda_x$? The following lemma answers this question (see Appendix for proof).

**Lemma 3.** Suppose that the level of fixed foreign-market-entry costs is equal to or greater than the level of fixed overhead production costs (i.e., $f_x \geq f_d > 0$). Then an increase in the number of trading partners $(n \uparrow)$ increases the export cutoff quality level $\lambda_x$; whereas a reduction in per-unit transport costs $(\tau \downarrow)$ or a reduction in foreign-market entry costs $(f_x \downarrow)$ decreases $\lambda_x$ (i.e., $d\lambda_x/dn > 0$, $d\lambda_x/d\tau < 0$, and $d\lambda_x/df_x > 0$).

To get intuition behind these results, first notice that the zero-profit cutoff condition implies that $(1 - \tau \lambda_x^{-1})L/M_c = f_x$, where $E=1$ is the equilibrium value of per-capita consumption expenditure. Obviously, an increase in $n$ raises the mass of varieties consumed $M_c$ (from Lemma 2 and (14)). This reduces the profits of the marginal exporter, and therefore it requires an increase in the export cutoff quality level $\lambda_x$ to restore profits from exporting back to zero. In other words, trade liberalization generates an income-based adverse demand effect that reduces the profits of all surviving firms including the marginal firms that earn zero profits. The zero-profit conditions that define the cutoff quality levels are restored only if the cutoff quality levels rise.

A decline in (variable or fixed) trade costs has two effects: It increases profits from exports (direct supply-based effect) and this requires a decline in the cutoff level to restore profits back to zero; and increases the number of available varieties.
(the indirect income-based demand effect as described above) which tends to reduce profits and requires an increase in the cutoff quality level to bring profits back to zero. The direct effect dominates (which is desirable for stability purposes), and consequently, lower foreign-market entry costs or lower trade costs generate lower export cutoff quality level \( \lambda_x \).

The next step of the analysis is to establish the impact of trade liberalization on markups and welfare. Lemma 2 and Eq. (16) ensure that any type of trade liberalization increases the domestic cutoff quality level, and subsequently raises the average domestic markup.\(^{17}\) The effects of trade liberalization on the average export markup depend on the nature of trade liberalization: an increase in the number of trading partners raises the average export markup; a reduction in foreign-market entry costs reduces the average markup; and a decline in per-unit trade costs has an ambiguous effect on the average export markup.\(^{18}\) In addition, inspection of Eq. (14) and Lemma 2 establish that any form of trade liberalization increases the domestic cutoff quality level \( \lambda_d \) and the mass of varieties available for consumption \( M_t \) in each country. Finally, differentiating equation (17) with respect to varieties consumed yields \( \partial U / \partial M_t = \ln(\beta / M_t) - 1 > 0 \), where the inequality follows from the love-for-variety assumption that requires a sufficiently large value of parameter \( \beta \) (see footnote 16). Thus trade liberalization raises national and global welfare. The following proposition summarizes these results.

**Proposition 3.** Suppose that the level of fixed foreign-market-entry costs is equal to or greater than the level of fixed overhead production costs (i.e., \( f_x = f_d > 0 \)). Trade liberalization

a. increases the average domestic markup (\( \mu_d \uparrow \));

b. has a differential effect on the average export markup (\( \mu_x \downarrow \)): an increase in the number of trading partners raises the average export markup; a reduction in foreign-market-entry costs lowers the average export markup; and a decline in per-unit trade costs has an ambiguous effect on the average export markup;

A move from autarky to (restricted) trade. In autarky, the cutoff quality level \( \lambda_d^A \) is determined by \( H(\lambda_d, 1) = \delta f / f_d \), and is strictly less than the open-economy cutoff quality level \( \lambda_d \): the absence of export markets reduces the benefits of entry and shifts labor from the production of higher-quality products towards the production of lower-quality products. Observe that Eqs. (14), (16), and (17) determine the closed-economy mass of varieties consumed \( M_t^A = M_t^u = \{1 - (\lambda_d^A)^{-1} \ln(f_d / f_t) \} \), the average markup \( \mu^A_d = \tau_d \), and the level of welfare \( U^A = \max_{\lambda_d} \ln(\beta / M_t) \) which are all functions of the domestic cutoff quality level.

**Proposition 4.** Suppose that foreign-market entry fixed cost is equal or greater than the fixed overhead production cost (i.e., \( f_x = f_d > 0 \)). A move from autarky to trade

a. increases the domestic cutoff quality level (\( \lambda_d > \lambda_d^A \)) and generates higher domestic markups (\( \mu_d > \mu_d^A \));

b. raises the mass of products available for consumption (\( M_t > M_t^A \)); and has a positive effect on national and global welfare (\( U > U^A \)).

The model’s mechanism that channels the effects of trade liberalization and the results complement the work of Melitz (2003) and Melitz and Ottaviano (2008) in several ways. Consider first the channel through which trade affects the economy. In the model proposed by Melitz (2003), trade liberalization operates by raising the real wage and thus forcing less productive firms to exit. This labor market channel induces a reallocation of resources from less productive to more productive firms. In Melitz and Ottaviano (2008), the sole mechanism that transmits the effects of trade works through the price elasticity of residual demand. Changes in the price elasticity of demand affect firm-level markups, prices and welfare. In the present model, although the real wage is allowed to change, the dominant mechanism operates through income (as opposed to price) changes. In other words, trade shifts leftward the unitary-elastic residual demand for each firm and forces low-quality firms out of the market.

Consider next the effects of trade liberalization on average domestic and export markups. In Melitz (2003), trade liberalization does not have any effects on markups because they depend only on the constant price elasticity of demand. In Melitz and Ottaviano (2008) trade intensifies the competition and reduces average (and firm-specific) domestic and export markups. In contrast, in our model trade increases the average domestic markup by forcing low-quality and low-price firms out of the market. The effects of trade on average export markups depend on the nature of trade expansion. An expansion in market size measured by the number of trading partners raises the average export markup, whereas a reduction in trade costs has an ambiguous effect on export markups. Interestingly, a reduction in foreign market entry costs reduces the average export markup as in Melitz and Ottaviano (2008) who abstract from this case by assuming that these costs are zero.

\(^{17}\) Differentiating the first equation of (16) with respect to \( \lambda_d \) implies \( d\lambda_d / d\lambda_d = (\lambda_d - \lambda_d) g(\lambda_d) / [1 - G(\lambda_d)] > 0 \) since \( \lambda_d < \lambda_d \). This, combined with Lemma 2, implies the above result.

\(^{18}\) Differentiating \( \mu_x \) with respect to \( \tau \) yields \( \partial \mu_x / \partial \tau = -1 / \delta \lambda_d \), implying \( \tau = \delta \lambda_d / \lambda_d \). The sign of the first term on the right hand side is positive since \( d\lambda_d / d\lambda_d > 0 \) and Lemma 3. Thus, a reduction in \( \tau \) has an ambiguous effect on the average export markup.
The BEJK model and its extension by de Blas and Russ (2010) deliver an endogenous distribution of markups in a Ricardian world with Bertrand competition. In both models, the total number of varieties in the world is fixed and exogenous. However, the possibility of free-entry in the de Blas and Russ model generates an endogenous number of potential entrants in each industry. In this case, trade liberalization increases efficiency directly by inducing more entry. An increase in the number of rivals squeezes markups by intensifying the competition among potential entrants and the incumbent firm. A higher number of rivals results in lower average markups. Although trade liberalization encourages more entry in our model too, the new entrants are not rivals of the existing firms. Each firm still retains certain monopoly power in the market, and the average markups change as the low-quality firms exit the market. In other words, in our model trade liberalization works through more intense market share competition among firms producing different varieties, whereas in their model trade intensifies competition across potential entrants competing within each product.

Another recent strand of literature has analyzed the pro-competitive effects of trade on welfare occurring through lower firm-specific markups (e.g., Edmond et al., 2012; Holmes et al., 2012). These models assume a nested pair of constant elasticity demand system and a fixed number of competitors producing each variety and competing in a Cournot fashion. Each oligopolist faces a variable demand elasticity which depends on its market share and charges an endogenous markup. A reduction in trade costs increases both the extensive and intensive margins of trade. Accordingly, trade has a strong downward impact on average markups and increases welfare.

In our model prices are exogenous reflecting quality differences across firms and Bertrand competition. Recall that each exporter charges a price \( p^x_i(x) = \lambda_i^{1+\pi} \) which is independent of variable trade costs, and incurs a marginal cost \( c_x^r = \tau x^r \). Its corresponding markup is \( \lambda_i/\tau \). This means that a reduction in variable trade costs does not affect its export price and quantity; but increases each firm's markup and profits. As a result, a reduction in trade cost does not affect the intensive margin. Higher firm profits induce more entry into the export market and increase the extensive margin. In summary, trade liberalization in our model works exclusively through the selection mechanism (i.e., the extensive margin), and therefore has a moderate effect on markups and welfare. Changes in firm-specific or average export markups do not affect the intensive margin.

How do our model's predictions about markups and trade liberalization relate to the empirical literature? Although the majority of the empirical studies has found that trade liberalization lowers markups, the jury is still out on the effect of trade on markups. For example, Levinsohn (1993) shows that the 1984 trade liberalization in the Turkish manufacturing sector decreased the markups in all five industries that he considers. Using calibration and simulation techniques, some recent studies (e.g., de Blas and Russ, 2010; Edmond et al., 2012; Arkolakis et al., 2012; Holmes et al., 2012) also find that trade lowers markups. On the other hand, using plant-level data from Cote d'Ivoire, Harrison (1994) finds that markups liberalization improves national and global welfare.

In addition, our model offers predictions regarding the effects of trade liberalization on the variance of markups. Under the assumption that the quality levels drawn from a Pareto distribution with a finite variance, the variance of markups is an increasing function of the average markup. Thus trade liberalization increases the variance of domestic markups and has an ambiguous effect on the variance of export markups. This prediction is consistent with suggestive evidence presented by Epifani and Gancia (2011). Based on US annual time series data for the period 1958–1996, the authors show that the dispersion of domestic markups is positively related to trade openness. More theoretical and empirical work is needed to understand better the nexus between trade openness and markups.

Despite these aforementioned differences, all theoretical models deliver similar predictions on the effect of trade on welfare in the case of symmetric countries. In the present model trade drives low-quality products out of the market, expands the mass of varieties consumed and increases national and global welfare. These results are derived in a market environment where firms face unitary-elasticity demand curves and the threat of imitation, engage in Bertrand price competition, charge heterogeneous markups, and face positive fixed costs. By arriving at the same conclusion, our model highlights the robustness of one of the main results of the theory of trade with heterogeneous firms: multilateral trade liberalization improves national and global welfare.

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19 Using data from a panel of European Union manufacturing sectors between 1989 and 1999, Chen et al. (2009) investigate the testable implications of the Melitz and Ottaviano model. They find that trade openness lowers prices and markups, but increases the productivity in the short-run. However, they also find that the long-run effects of trade openness are ambiguous and may even be anti-competitive.

20 Badinger (2007) found that the single market programme in Europe reduced domestic markups in the manufacturing and construction sectors but increased markups in the service industries.

21 Assume that the Pareto distribution that governs the quality levels is given by \( G(\xi) = 1 - \xi^{-k} \) where \( k > 2 \) is a parameter that captures the dispersion of the quality levels. In this case, the average domestic and export markups are given by \( \mu_d = k_d/(k-1) \) and \( \mu_e = k_e/[k(k-1)] \), respectively, where \( \lambda_d \) and \( \lambda_e \) are the domestic and export cutoff quality levels. The corresponding variances are given by \( \sigma_i^2 = \mu_i^2/[k(k-2)] \) for \( i = d, x \).
4. Spatial considerations

This section augments the original model by adding spatial cost considerations. Consider now the case where both trade costs and imitation costs increase with geographic distance. In this modified version of the model, firms with higher-quality products enjoy higher market shares, sell their products in more markets, and charge higher prices in far-away markets.

We introduce spatial elements by considering a global economy with four countries located in a circle. Each country faces two adjacent (neighboring) countries and a distant country. Thus, if a home is indexed by 0, it has two neighbors (1 and 3), country 1 has two neighbors (1 and 2), and so on. Let $l_{t}$ denote the level of trade costs between two adjacent countries, and $\phi_{t}$ denote the level of trade costs between two distant countries, where $\phi > 1$.

As in the original model, the marginal cost of producing a variety with $\lambda$ quality is $c(\lambda) = \lambda^{q}$ and the quality of a generic (imitated) product is $\lambda_{0} = 1$. In addition, suppose that a firm’s ability to imitate a product depends on geographic distance.

Specifically, we assume that the marginal costs of producing an imitated product depends on the quality of the original product $\lambda$. Thus the marginal costs of producing an imitated product with quality $\lambda$ is $c(\lambda) = \lambda^{q}$, if the potential imitator resides in the same country where the product is discovered; $\phi_{t}\lambda^{q}$, if potential imitator resides in an adjacent country; and $\phi_{t}^{2}\lambda^{q}$, if a potential imitator resides in the distant country. Independently of the marginal costs of production, the quality of generic products is equal to unity. The original model can be recovered by setting $\phi = 1$ and considering $n$ instead of three trading partners.

Given these assumptions, a home firm that discovers a variety with quality $\lambda$ follows a limit-pricing strategy and charges a price $p_{d}(\lambda) = \lambda^{q}$ in the domestic market, $p_{d}(\lambda) = \phi_{t}\lambda^{q}$ in an adjacent-country market; and $p_{d}(\lambda) = \phi_{t}^{2}\lambda^{q}$ in the distant-country market. Therefore, firms with higher quality products charge higher prices in each market; and conditional on the level of quality, each firm’s price increases with distance from the origin of production.

Cost heterogeneity implies that per-period profits of firms that export to all three countries can be decomposed into three parts: profits from domestic sales $\pi_{d}(\lambda)$; profits from exports into each of the two adjacent countries $\pi_{a}(\lambda)$; and profits from exports into the distant market $\pi_{f}(\lambda)$.

\[
\pi_{d}(\lambda) = [p_{d}(\lambda) - \lambda^{q}]q_{d}(\lambda) - f_{d} = (1 - \lambda)^{-1} \frac{EL}{M_{c}} - f_{d},
\]

\[
\pi_{a}(\lambda) = [p_{a}(\lambda) - \phi_{t}\lambda^{q}]q_{a}(\lambda) - f_{a} = (1 - \phi_{t}\lambda)^{-1} \frac{EL}{M_{c}} - f_{a},
\]

\[
\pi_{f}(\lambda) = [p_{f}(\lambda) - \phi_{t}^{2}\lambda^{q}]q_{f}(\lambda) - f_{f} = (1 - \phi_{t}^{2}\lambda)^{-1} \frac{EL}{M_{c}} - f_{f},
\]

where subscripts $a$ and $f$ denote adjacent and distant (far-away) markets.

According to Eqs. (18a)–(18c), the flow of profits in each market is an increasing and monotonic function of quality $\lambda$. Because $\pi_{d}(1) = -f_{d} < 0$, $\pi_{d}(\phi_{t}) = -f_{d} < 0$, and $\pi_{f}(\phi_{t}^{2}) = -f_{f} < 0$, there exist three cutoff quality levels $\lambda_{a}$, $\lambda_{o}$, and $\lambda_{f}$ such that $\pi_{d}(\lambda_{a}) = \pi_{a}(\lambda_{f}) = \pi_{f}(\lambda_{f}) = 0$. These ex-post zero-profit conditions lead to the following expressions:

\[
\lambda_{a} = \frac{\phi_{t}}{1 - (1 - \lambda_{a})^{k}f_{d}/f_{a}} \quad \text{and} \quad \lambda_{f} = \frac{\phi_{t}^{2}}{1 - (1 - \lambda_{f})^{k}f_{f}/f_{a}}.
\]

As in the simple framework, there are three remarks. First, the requirement that the denominator of these equations must be non-negative implies that $\lambda_{a} \in (1, k)$, where $k = \min(1/(1 - f_{d}/f_{a}), 1/(1 - f_{f}/f_{a}))$. Second, equations in (19) indicate that $\lambda_{a}$ and $\lambda_{f}$ are increasing monotonically in $\lambda_{a}$. Third, under the standard parameter restrictions $f_{f} > f_{d} > f_{a} > 0$ (i.e., foreign market entry costs increase with distance from home market), $\tau > 1$ and $\phi > 1$, we have $\lambda_{f} > \lambda_{o} > \lambda_{a} > 1$, and $k = 1/(1 - f_{d}/f_{a})$. Therefore the set of firms in each country is partitioned into three groups: firms whose product quality level is less than $\lambda_{a}$ exit the market; firms whose product quality level is $\lambda \in (\lambda_{a}, \lambda_{f})$ produce exclusively for the domestic market; firms whose product quality is $\lambda \in (\lambda_{f}, \lambda_{a})$ serve the domestic and each of the two adjacent markets; and firms producing products with $\lambda \geq \lambda_{f}$ sell their products in all markets.

The ex-post distribution of product quality levels $\psi$ is given by (10). The probabilities that a surviving firm exports to an adjacent and distant countries respectively are given by

\[
\zeta_{a} = \frac{1 - G(\lambda_{a})}{1 - G(\lambda_{a})} \quad \text{and} \quad \zeta_{f} = \frac{1 - G(\lambda_{f})}{1 - G(\lambda_{a})}.
\]

Thus, the measure of home firms serving a particular market decreases in distance $\phi$. It follows straightforwardly to show that the average per-period flow of profits of a successful entrant is now given by

\[
\pi = (1 - \lambda_{a})^{2} \frac{EL}{M_{c}} - f_{d} + 2\zeta_{a} \left(1 - \phi_{t}\lambda_{a}^{-1}\right) \frac{EL}{M_{c}} - f_{a} + \zeta_{f} \left(1 - \phi_{t}^{2}\lambda_{f}^{-1}\right) \frac{EL}{M_{c}} - f_{f}.
\]

---

22 We still assume that the technology to produce generic products diffuses internationally through imports. The present way of modeling imitation captures the idea that the degree of technology transfer declines with distance. See, for instance, Keller (2002) for evidence on the local nature of international knowledge spillovers.
where \( \tilde{\lambda}_i \) represents the weighted harmonic mean of quality levels in each market. Using this average profit function in the free-entry condition (11) yields

\[
f_d H(\lambda_d,1) + 2 f_a H(\lambda_a,\phi \tau) + f_f H(\lambda_f, \phi^2 \tau) = 0,
\]

where \( H() \) is defined by (13) and is strictly decreasing in its first argument as shown in Appendix. Equations in (19) and (20) determine the unique values of the three cutoff quality levels and can be used to perform the standard comparative statics exercises.

The combination of limit-pricing strategies, Cobb–Douglas preferences and cost heterogeneity generates heterogeneous market shares, prices, and markups consistent with the findings of several recent empirical studies (Baldwin and Harrigan, 2011; Verhoogen, 2008; Kugler and Verhoogen, 2012). In this modified model, firms with higher-quality products face higher marginal cost of manufacturing; charge higher prices in more distant markets; and enjoy higher markups, larger revenues and market shares. More firms export to more proximate markets, and sufficiently high foreign market entry costs eliminate bilateral trade flows between distant markets.

5. Welfare properties

The constrained optimality of the cutoff productivity levels has been demonstrated in a multi-sector version of the Melitz model by Feenstra and Kee (2008). This section addresses a similar question in the context of the present model: is the market equilibrium socially optimal? Since trade liberalization increases national and global welfare, the reader might be tempted to conclude that the market achieves the first-best solution here as in Melitz (2003). Unfortunately, this conclusion could be misleading. The present model generates the possibility of divergence between the socially optimal and laissez-faire equilibrium due to the endogenous distribution of markups.

In order to minimize the algebra, we illustrate this possibility in a closed-economy setting, noting that similar considerations apply to an open economy, where trade costs and foreign-market entry costs create additional welfare distortions. The social planner maximizes per-capita utility function \( U = M_c \ln(\beta E/M_c) \) with respect to the domestic cutoff quality level \( \lambda_d \), per-capita expenditure \( E \) and the mass of varieties available for consumption \( M_c \), subject to the labor market clearing condition

\[
\frac{\delta M_c}{1-G(\lambda_d)} + f_d M_c + \tilde{\lambda}_d^{-1} E L = L,
\]

where the first term on the right-hand-side is the total labor employed by potential entrants, and \( f_d M_c + \tilde{\lambda}_d^{-1} E L \) is the total labor employed in production \( (L_p = \int_p f_d + q(\lambda))M_c(\psi(\lambda)) d\lambda = f_d M_c + \tilde{\lambda}_d^{-1} E L \). The constraint optimization yields the following first-order conditions:

\[
(\lambda_d^{-1} - \tilde{\lambda}_d^{-1})(E/M_c) = \delta f_a/[1-G(\lambda_d)],
\]

\[
[\tilde{\lambda}_d^{-1} \ln(\beta E/M_c) - \lambda_d^{-1}](E/M_c)L = f_d.
\]

Since the average quality level \( \lambda_d \) is a function of the cutoff quality level \( \lambda_d \), Eqs. (22a) and (22b) determine the socially optimal values of \( \lambda_d^* \) and \( E^* \). Once these two endogenous variables are determined, the resource constraint (21) can be used to obtain individual values for \( E^* \) and \( M_c^* \).

How does the socially optimum solution compare to the closed-economy market equilibrium? Set the number of trading partners \( n \) equal to zero in (12) and substitute the resulting expression for \( \Pi \) and \( f_d \) into (11) to obtain the closed-economy free-entry condition which is identical to (22a). Thus, the market equilibrium values \( \lambda_d^* \) and \( E^*/M_c^* \) are simultaneously determined by the entry condition (22a) and the zero-profit condition (5). The latter is reproduced below for illustrative purposes

\[
[1-\lambda_d^{-1}](E/M_c)L = f_d.
\]

Comparing the zero-profit condition (23) to the socially-optimal condition (22b) reveals the following. The market cutoff quality level depends on profitability considerations that are captured by the marginal price-cost margin \( 1-\lambda_d^{-1} \). This differs from the efficient price-cost margin \( \lambda_d^{-1} \) in (22b) which depends on the ratio between the marginal utility derived from an additional variety and the average quality of available products. Thus, the planner cares about the average (infra-marginal) consumer, whereas the market cares about the marginal consumer. On one hand, because the marginal quality level is less than the average quality level \( \lambda_d < \lambda_d \), the market has a tendency to overstate the benefits of introducing a new variety by charging a lower price and producing a higher quantity than the social planner. On the other hand, because \( \ln(\beta E/M_c) > 1 \) and \( \lambda_d > 1 \) the ranking between the social and market prices is in general ambiguous.

23 All our results go through in the case where markets are interpreted as distinct geographic regions within a particular country and trade costs as transportation costs. This interpretation is consistent with the existence of heterogeneous prices, markups and market shares across firms within a particular country.

24 The second equation in (19) implies that, for a sufficiently high value of foreign market entry cost \( f_f \), the cutoff quality level \( \lambda_f \) becomes infinite. In this case, there is no bilateral trade between two distant countries.
Proposition 5. The laissez-faire cutoff quality level $\lambda_d$, mass of varieties $M_d$, and per-capita expenditure $E^A$ are socially sub-optimal.

What are the implications of the suboptimal market equilibrium for the welfare effects of trade liberalization? In the present model, social welfare depends positively on the mass of consumed varieties $M_c$ and expenditure per variety $E/M_c$. Notice that for any level of expenditure per variety $E/M_c$, the laissez faire cutoff quality level $\lambda_d$ is socially efficient. This result, which is identical to the one obtained by Feenstra and Kee (2008), holds because the cutoff quality level does not appear as an argument in the social planner's objective function, but only in the resource constraint.

The social planner can obtain the first-best solution by simultaneously choosing the cutoff quality level to obtain an efficient allocation of resources and the level of per-capita expenditure to adjust optimally the term $E/M_c$. The optimal level of the cutoff quality level can be achieved by appropriate production subsidies or taxes that target the price-cost margin $1 - \lambda_d^{-1}$ or the fixed production cost $f_d$. In addition, the government can use wage subsidies or taxes to adjust the term $E/M_c$.

Under the market, the presence of imitation introduces a price-based consumption distortion which is reflected in the suboptimal average markup. In addition, free entry into the research sector and the absence of savings implies that $E = w = 1$. Thus the market has only one instrument, the cutoff quality level (which determines the mass of consumed varieties) to affect the level of welfare. An increase in the domestic cutoff quality level raises the mass of varieties consumed and affects welfare through two distinct channels. First, it reduces the expenditure per variety $1/M_c$ and the “average quantity” consumed. Second, it increases the level of welfare directly because the latter is proportional to the mass of consumed varieties. The assumption of a sufficiently large $\beta$ ensures that the latter effect dominates. In other words, for each level of per-capital consumer expenditure, welfare increases in the mass of varieties consumed. Thus a trade-induced expansion of varieties consumed increases welfare through the “love-for-variety” effect even if income-related distortions are present. These distortions can be reduced or eliminated through domestic policies that increase per-capita expenditures and raise the welfare level further. Consequently, the presence of distortions does not reverse the variety-based welfare gains from trade.

We suspect that relaxing the assumptions of structurally identical countries might generate the possibility of welfare-reducing trade. However, the incorporation of asymmetric countries and more sectors would complicate considerably the mathematical structure of the model, and thus it is beyond the scope of the present paper.

6. Concluding remarks

The present paper developed a tractable model of quality heterogeneity and international trade. In our model, firms with high-quality products export, firms with intermediate-quality products produce for the domestic market, and firms with low-quality products exit the market. Firms producing high-quality products charge high prices and enjoy high markups, whereas firms producing low-quality products charge low prices and charge low markups.

The model generates several novel insights. First, trade liberalization benefits more firms with higher-quality products, forces inefficient firms to exit, and raises the average domestic markup. The effects of trade liberalization on average export markups are in general ambiguous and depend on the nature of trade liberalization. Second, trade liberalization increases the mass of varieties available for consumption and raises national and global welfare. Third, despite the positive welfare effect of trade liberalization, there is more room for further welfare improvements because the laissez-faire equilibrium is suboptimal. Finally, our framework can be easily augmented to incorporate cost heterogeneity and spatial elements. In the modified version of the model, firms with higher-quality products enjoy higher market shares, sell their products in more markets and charge higher prices in far-away markets. These predictions are consistent with the findings of several recent empirical studies.

The model is flexible enough to address several other interesting issues in the trade literature. For example, embedding this model into the Heckscher–Ohlin model and investigating the implications of trade on productivity, markups, and welfare is an interesting extension. Bernard et al. (2007) has conducted similar experiment with the Melitz model, and they show that exposure to trade rises aggregate productivity in all sectors, but the productivity improvement is bigger in comparative advantage sectors. Similarly, our preliminary analysis suggest that trade openness rises average quality in all sectors, but the quality improvement is stronger in comparative advantage sectors. This further ensures that exposure to trade increases domestic markups in all sectors, but it has stronger impact on the domestic markups in comparative advantage sector.

A second generalization is to relax the assumption of two identical countries and develop a North–South model of trade and quality heterogeneity. Such a model could address several novel issues including patterns of vertical trade, North–South income inequality, the effects of asymmetric trade liberalization, and the nexus between per-capita income and markups.

Finally, a third interesting extension is to incorporate horizontal FDI into our model. Assuming that foreign-market-entry fixed costs associated with FDI exceeds the fixed exporting cost as in Helpman et al. (2004), the set of incumbents will be divided into three groups: firms with low quality products produce exclusively for the domestic market; firms with

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25 The zero profit condition (23) implies that any of the aforementioned policy that increases profits such as a production subsidy or a subsidy that reduces fixed production costs yields a lower cutoff quality level.

26 Dinopoulos and Unel (2011) extends this model to a product innovation endogenous growth model, and they show that exposure to trade has an ambiguous effect on economic growth.
intermediate quality products sell their products in both the domestic and foreign markets via exporting; and firms producing at the high end of the quality spectrum serve the domestic market and engage in FDI. Based on the analysis of Section 3 and the insights of Helpman et al. (2004), we conjecture that allowing horizontal FDI would increase average domestic and export markups by forcing inefficient domestic and exporting firms producing low-quality products to exit. These issues constitute fruitful avenues for future research.

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Appendix A. Limit pricing

Consider the following utility function, which is a generic version of the preferences used routinely in quality-ladders growth models, that describes the tastes of a typical consumer

\[ U = \int_{\Omega} \ln \left[ \beta q_0 + \beta \lambda(0) \frac{q(0)}{L} \right] d\Omega, \]

where \( \Omega \) denotes the set of potential varieties. Each variety is associated with two quality levels: \( q(\lambda(0)) \) that can be produced by only one firm as in our model, and the low-quality (generic) version \( q_0 \) that can be produced by a competitive fringe after the original product has been produced. The functional form of \( U \) implies that the elasticity of substitution across varieties is unity. It also implies that low-quality and high-quality versions of a good are identical (perfect substitutes) when adjusted for quality: one unit of high-quality good \( q \) generates the same utility as \( \lambda \) units of the low-quality good \( q_0 \). As a result, the elasticity of substitution between different quality versions of a typical good is infinite.

We assume that the low-quality version can be produced under perfect competition as described in the paper. This implies that the generic product commands a price \( p_0 = \lambda^0 \). In addition, since both goods adjusted for quality are identical, a consumer spends her income only on the good with the lower quality adjusted price. Thus, if a consumer buys only the generic good \( q_0 \), she obtains a utility level equal to \( \ln(\beta E(p_0M,L)) = \ln(\beta E(M,L)) \), whereas if she buys the high-quality good she obtains a utility level equal to \( \ln(\beta E(p_\lambda M,L)) \).

Therefore the consumer is indifferent between the two quality versions of the product if and only if \( p(\lambda) = \lambda^{1+\theta} \). If the high-quality producer sets a limit price \( p = \lambda^0 \), where \( \lambda^0 \) is infinitesimally small, she maximizes profits and drives the competitive fringe out of the market. Following the standard practice of quality-ladders growth models, we assume that even if \( \lambda^0 = 0 \), all consumers buy the high-quality version of each product even if in principle they are indifferent between the two quality versions of each good.

Appendix B. Proof of Proposition 2

We will prove that \( H(\lambda_i , x) \) defined in (13) is strictly decreasing in \( \lambda_i \). Let \( J(\lambda_i, x) = [1 - G(\lambda_i)](1 - x\lambda_i^{-1}) \). Since \( \lambda_i > x \), it easily follows that \( J(\lambda_i, x) > 0 \). Applying the Leibnitz rule in calculus, we have

\[ dJ(\lambda_i, x)/d\lambda_i = -(1 - x\lambda_i^{-1})g(\lambda_i). \]

Differentiating \( H(\lambda_i, x) \) with respect to \( \lambda_i \) yields the desired result

\[ \frac{\partial H}{\partial \lambda_i} = (1 - x\lambda_i^{-1}) \frac{dJ/d\lambda_i - x\lambda_i^{-3} J}{1 - x\lambda_i^{-1}} + g(\lambda_i) = \frac{-x\lambda_i^{-2} J}{(1 - x\lambda_i^{-1})^2} < 0. \]

The property \( \partial H/\partial \lambda_i < 0 \) in conjunction with equations in (6) imply that the left hand side of (12) is strictly decreasing in the cutoff quality level \( \lambda_i \).

The full characterization of the solution requires the evaluation of the left-hand-side of (12) when \( \lambda_d \) approaches its boundaries. The following conditions characterize the limiting behavior of \( H(\lambda_d, 1) \) and \( H(\lambda_\infty, \tau) \):

\[ \begin{align*}
\lim_{\lambda_d \to 1} H(\lambda_d, 1) &= \infty, & \lim_{\lambda_d \to 1} H(\lambda_\infty, \tau) &= \infty, \\
\lim_{\lambda_d \to k} H(\lambda_d, 1) &= 0, & \lim_{\lambda_\infty \to 0} H(\lambda_\infty, \tau) &= 0.
\end{align*} \tag{24a} \tag{24b} \]

To prove the claims in (24a), first note that \( J(\lambda_i, x) = \int_{\lambda_i}^{x}[1 - x\lambda_i^{-1}]g(\lambda) d\lambda \). Inserting this expression into \( H(\lambda_i, x) \) yields

\[ H(\lambda_i, x) = \int_{\lambda_i}^{x}[1 - x\lambda_i^{-1}]g(\lambda) d\lambda / \left(1 - x\lambda_i^{-1}\right) = \left[1 - G(\lambda_i)\right]. \]
The numerator of the first term on the right hand side is always positive as long as \(g(\cdot)\) is continuous at \(\lambda_i\). The claims in (24a) easily follow as we take the limits of both sides as \(\lambda_d \to 1\) and \(\lambda_x \to \tau\). Also, note that as \(\lambda_x \to \infty\), the right hand side of the above equation approaches to 0, i.e., \(\lim_{\lambda_x \to \infty} H(\lambda_x, \tau) = 0\). The first claim in (24b) immediately follows from \(H(k, 1) = (1 - G(k))(k^{-1} - \tilde{k})/(1 - k^{-1}) > 0\) since \(k^{-1} > \tilde{k}^{-1}\), where \(\tilde{k} = 1/(1 - G(k)) f_k^\infty \lambda_i g(\lambda_i) d\lambda_i^{-1}\).

In summary, the left-hand-side of (12) is always positive and defines a negatively sloped curve starting at infinity as \(\lambda_d \to 1\) and reaching the value \(H(k, 1)\) as \(\lambda_d \to k\). Thus, a sufficient but hardly necessary condition for the existence of the unique equilibrium is that \(H(k, 1) < \partial f_x/\partial d\). It is then obvious from equations in (6) that the unique domestic cutoff quality level \(\lambda_d\) determines the unique export cutoff quality level \(\lambda_x\) such that \(\lambda_x > \tau\) and \(\lambda_x > \lambda_d\).

Appendix C. Proof of Lemma 2

Totally differentiating Eqs. (6) and (12) yields

\[
\frac{d\lambda_d}{dn} = -H_x \left( f_x \frac{\partial H_d}{\partial \lambda_d} \right) \left[ \frac{\partial \lambda_x}{\partial \lambda_x} + \frac{n}{\tau} \left( f_x \frac{\partial \lambda_x}{\partial \lambda_x} \right)^2 \frac{\partial H_x}{\partial \lambda_x} \right] > 0, \tag{25a}
\]

\[
\frac{d\lambda_x}{d\tau} = -nf_x \left[ \frac{\partial \lambda_x}{\partial \lambda_x} \frac{\partial H_x}{\partial \lambda_x} + \frac{\partial H_x}{\partial \lambda_x} \frac{\partial \lambda_x}{\partial \lambda_x} \right] \left[ \frac{\partial \lambda_x}{\partial \lambda_x} + \frac{n}{\tau} \left( f_x \frac{\partial \lambda_x}{\partial \lambda_x} \right)^2 \frac{\partial H_x}{\partial \lambda_x} \right] > 0, \tag{25b}
\]

\[
\frac{d\lambda_d}{df_x} = [1 - G(\lambda_x)] \left[ \frac{\partial \lambda_x}{\partial \lambda_x} + \frac{n}{\tau} \left( f_x \frac{\partial \lambda_x}{\partial \lambda_x} \right)^2 \frac{\partial H_x}{\partial \lambda_x} \right] < 0. \tag{25c}
\]

where \(H_d = H(\lambda_d, 1)\) and \(H_x = H(\lambda_x, \tau)\). The sign of the last bracket in each expression is negative due to Proposition 2 which also implies that the sign of the first bracket in (25b) is negative. To see this note that the first term of that bracket can be written as

\[
\frac{\lambda_x \frac{\partial H_x}{\partial \lambda_x}}{\tau \frac{\partial \lambda_x}{\partial \lambda_x}} = \frac{[1 - G(\lambda_x)](1 - \lambda_x)^{-1}}{\lambda_x(1 - \lambda_x)^{1/2}},
\]

and that the second term in the same bracket is

\[
\frac{\partial H_x}{\partial \tau} = \frac{[1 - G(\lambda_x)] \left[ \lambda_x^{-1} - \lambda_x^{-1} \right]}{1 - \lambda_x^{-1}}.
\]

Adding these two expressions yields

\[
\frac{\lambda_x \frac{\partial H_x}{\partial \lambda_x}}{\tau \frac{\partial \lambda_x}{\partial \lambda_x}} + \frac{\partial H_x}{\partial \tau} = \frac{[1 - G(\lambda_x)] \lambda_x^{-1}}{1 - \lambda_x^{-1}} < 0.
\]

Appendix D. Proof of Lemma 3

First, totally differentiating (6) with respect to \(n\) yields

\[
\frac{d\lambda_x}{dn} = \frac{1}{\tau} \left( \frac{\lambda_x}{\lambda_d} \right)^2 \frac{d\lambda_d}{dn} > 0,
\]

where the inequality follows from (25a) in the previous section. Totally differentiating (12), on the other hand, yields

\[
\frac{d\lambda_x}{d\tau} = -\left[ \frac{\partial H(\lambda_d, 1)}{\partial \lambda_d} \frac{d\lambda_d}{d\tau} + \frac{n}{\tau} \frac{f_x \frac{\partial H(\lambda_x, \tau)}{\partial \lambda_x}}{\partial \lambda_x} \right] \left[ \frac{n}{\tau} \frac{f_x \frac{\partial H(\lambda_x, \tau)}{\partial \lambda_x}}{\partial \lambda_x} \right]^{-1} > 0,
\]

\[
\frac{d\lambda_x}{df_x} = -\left[ \frac{\partial H(\lambda_d, 1)}{\partial \lambda_d} \frac{d\lambda_d}{df_x} + \frac{n}{\tau} \frac{f_x \frac{\partial H(\lambda_x, \tau)}{\partial \lambda_x}}{\partial \lambda_x} \right] \left[ \frac{n}{\tau} \frac{f_x \frac{\partial H(\lambda_x, \tau)}{\partial \lambda_x}}{\partial \lambda_x} \right]^{-1} > 0.
\]

References
