The Ancient and Medieval: China and India

Douglas Pfeffer
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Outline

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2. India
Civilizations along the Yangtze and Huanghe (or Yellow) rivers are comparable in age to those of the Nile and Tigris/Euphrates.
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- Unfortunately, however, far fewer mathematical texts have survived
- The majority of what we have are not original texts, but replicas
  - The oldest text we currently have is the Zhoubi Suanjing (Chou Pei Suan Ching)
Zhoubi Suanjing

The oldest of the mathematical classics, it is estimated to be from 1200 BCE or 100 BCE. Modern scholars put it to be written a little after 300 BCE near the Han dynasty (202 BCE).

Concerns astronomical calculations, the Pythagorean theorem, and fractions.

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Its demonstration of the Pythagorean theorem
The Nine Chapters

Jiuzhang Suanshu (or Nine Chapters on the Mathematical Arts)

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  - The final problem was one considering 4 equations and 5 unknowns
The Nine Chapters

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The very first recorded Magic Square is found in this treatise:

```
 2  7  6  15
 9  5  1  15
 4  3  8  15
15 15 15 15
```

Reportedly, this square was brought to man by a turtle from the River Luo in the days of the legendary Emperor Yii.

Fun fact: This is the smallest (and unique up to rotation and reflection) non-trivial case of a magic square, measuring 3 x 3.
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3x + 2y + z &= 39 \\
2x + 3y + z &= 34 \\
x + 2y + 3z &= 26
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It started by drafting up the following grid:

\[
\begin{pmatrix}
1 & 2 & 3 \\
2 & 3 & 2 \\
3 & 1 & 1 \\
26 & 34 & 39
\end{pmatrix}
\]
The Nine Chapters

Through tedious descriptions:

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\begin{bmatrix}
1 & 2 & 3 \\
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\end{bmatrix}
\rightarrow
\begin{bmatrix}
0 & 0 & 3 \\
0 & 5 & 2 \\
36 & 1 & 1 \\
99 & 24 & 39
\end{bmatrix}
\]

They finished the problem by back solving the equations
36z = 99, 5y + z = 24, and 3x + 2y + z = 39.
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Chinese numeration essentially consisted of two systems: the first (and less popular) was ‘multiplicative’. It had ciphers for 1−10 and then for powers of 10.

\[678 = 6 \times 10^2 + 7 \times 10^1 + 8 \times 10^0\]

The second (and more popular) was the so-called Rod Numerals. This system was positional and had ciphers for 1−9 and multiples of 10 up to 90. Numbers, as in the first system, were read in pairs.
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  - In a 1247 CE text the value 1,405,536 is given:
- This was the most common system and, since it was centesimal, was useful for computation.
The Abacus

Rod numerals weren’t just a notation for computation: Actual bamboo rods were carried about in a bag by administrators and used as calculation devices on ‘counting boards’.

So dexterous were these counters that an 11th century writer described them as “flying so quickly that the eye could not follow their movement.”

These counting boards anticipated the abacus, which are relatively new (c. 1500s), but the concept dates back to the 500s.
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The Chinese Remainder Theorem appears in the 3rd-century book *Sunzi Suanjing* by Sunzi. The problem stated is:

"There are certain things whose number is unknown. If we count them by threes, we have two left over; by fives, we have three left over; and by sevens, two are left over. How many things are there?"

Sunzi's work contains neither a proof nor a full algorithm. Much later, algorithms would be developed by Indian mathematicians Aryabhata (6th century) and Brahmagupta (7th century), and in Fibonacci's *Liber Abaci* (1202).
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Early Chinese works used various approximations for $\pi$:

3, 3.1547, $\sqrt{10}$, $\frac{22}{7}$, $\frac{142}{45}$

In 200 CE, Liu Hui reworked the *Nine Chapters* and, using a 96-gon, achieved $\pi \approx 3.14$. Then used a 3072-gon to get $\pi \approx 3.14159$

Of interest is that Liu Hui also, much as the Babylonians had much earlier, correctly calculated the volume of the frustrum of a right-pyramid. Oddly enough, when he tackled the frustrum of a cone, he just used $\pi = 3$. 

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This approximation was not matched until the 1400s. How Changzhi achieved this approximation is not known.
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- Primarily arithmetic and number theory
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From the 10th to the 13th centuries, no new mathematical breakthroughs seem to have occurred

- This is interesting since these centuries saw the invention of paper, the compass, and gunpowder
As the Sung dynasty ended, China saw the Mongol expansion and increased contact with Islam. Li Zhi of Peking (Beijing) was a hermit, scholar, and academician. He wrote *Ceyuan Haijing* (Sea-Mirror of the Circle Measurements), which contained 170 problems about circles inscribed within (or circumscribed without) a right triangle and determining the relationships between the sides and the radii.
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Of interest, however, is that these last two contributions are seldom attributed to Hui. Similar results were published in the more popular text *Precious Mirror* by Zhu Shiji.
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  - In 1303, he wrote *Siyuan yujian (Jade Mirror of the Four Origins)*
    - The four origins were heaven, earth, man, and matter. They represented the four unknown quantities in a given equation.
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\[ 1 + 8 + 30 + 80 + \ldots + \frac{n^2(n+1)(n+2)}{3!} = \frac{n(n+1)(n+2)(n+3)(4n+1)}{5!} \]
Jade Mirror of the Four Origins

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Also included a systematic study of binomial coefficients and exhibited what is now mistakenly attributed to Blaise Pascal (1600s)

- Featured Rod Numerals with the empty position $O$
- Zhu himself claims the triangle is ‘known’ and ‘old’ and, notably, not his discovery
Chinese ‘Pascals Triangle’
Closing on the Chinese scene

After the 13th century, mathematics in China declined back into commercial arithmetic and routine study of the Nine Chapters. We now turn our eyes westward toward the Indian subcontinent...

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Archaeological excavations at Mohenjo Daro and Harappa give evidence to an old and highly cultured civilization in the Indus Valley around 2650 BCE.

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The area, however, was volatile through movement and conquest. Even Indian languages were not entirely uniform.
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The area, however, was volatile through movement and conquest. Even Indian languages were not entirely uniform.

- The Vedas, a group of ancient religious texts, do give detailed building prescriptions for altars and the like
  - These prescriptions came in the form of the *Sulbasutras* or “rules of the chord”
  - Eerily similar to Egyptian geometry (though few scholars support a strong connection to Egypt due to a lack of continuity in Indian mathematics)
Sulbasutras

- Written by many authors (all in verse) as early as 1000 BCE
- Contains, in part, of Pythagorean triples (although there is little concrete evidence of Mesopotamian influence)
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  - Contains, in part, of Pythagorean triples (although there is little concrete evidence of Mesopotamian influence)
- One such author, Apastamba, gave a number of geometric arguments
  - For example, a construction on how to, given a rectangle, draft up a square with the same area
Aryabhata
In 499 CE, Aryabhata wrote *Aryabhatiya*, a text on astronomy and mathematics.
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- Written entirely in verse
- Although the names of Indian mathematicians exist before him, none of their work survived
  - *Aryabhatiya*, much like Euclid's *Elements*, was a summary of earlier developments compiled by a single author
  - Unlike *Elements*, however, *Aryabhatiya* exhibited no deductive methodology
The mathematics portion opened with the powers of 10 and rules on obtaining the square and cube roots of integers
Aryabhatiya

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- Other examples include:
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Other examples include:
- area of a circle vs. the volume of a sphere
- area of a trapezoid vs. area of an arbitrary plane figure
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The fact that Aryabhata may have been influenced by the Greeks is further supported by his adoption of the myriad as a unit length in geometry
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For example, when solving the equation \( \frac{a}{b} = \frac{c}{x} \) for \( x \) (supposing \( a, b, \) and \( c \) are known), he writes:

- “In the rule of three multiply the fruit by the desire and divide by the measure. The result will be the fruit of the desire.”
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- For example, when solving the equation \( \frac{a}{b} = \frac{c}{x} \) for \( x \) (supposing \( a, b, \) and \( c \) are known), he writes:
  - “In the rule of three multiply the fruit by the desire and divide by the measure. The result will be the fruit of the desire.”
- Here,
  - \( a = \) ‘measure’,
  - \( b = \) ‘fruit’,
  - \( c = \) ‘desire’, and
  - \( x = \) ‘fruit of the desire’
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Multiplication

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This method was hypothesized to be formulated c. 12th century in India and then disseminated to China and Arabia and from Arabia to Italy in the 14th and 15th centuries.
Brahmagupta
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- Lived in 7th century in Central India (about a century after Aryabhata)
  - In his works, he mentions two values for $\pi$
    - Practical value: $\pi \approx 3$
    - Neat value: $\pi \approx \sqrt{10}$

Notably, no mention to Aryabhata’s estimate – suggesting little inheritance.

Best known work was Brahmasphuta Siddhanta.

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- Brahmagupta systematized the arithmetic of negative numbers and the value zero
Brahmasphuta Siddhanta

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On the matter of zero, Brahmagupta seemed to have made a false start:

- He argued that \( \frac{0}{0} = 0 \), but on \( \frac{a}{0} \) for \( a \neq 0 \), he avoided it...
- “Positive divided by positive, or negative by negative, is affirmative. Cipher divided by cipher is naught. Positive divided by negative is negative. Negative divided by affirmative is negative. Positive or negative divided by cipher is a fraction with that for denominator”
Brahmagupta was apparently the first to provide a general solution to the linear Diophantine equation $ax + by = c$ where $a, b, c \in \mathbb{Z}$. He also suggested the quadratic Diophantine equation $x^2 = 1 + py^2$. This equation is often (mistakenly) attributed to the 1600s mathematician John Pell – the so-called Pell's equation. Special cases of Pell's equation were solved by the next prominent Indian mathematician Bhaskara in c. 1100.
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- Special cases of Pells equation was solved by the next prominent Indian mathematician Bhaskara in c. 1100.
Bhaskara improved on other aspects of Brahmagupta's work as well. In addressing the $a_0$ for $a \neq 0$ debacle, he writes:

"Statement: Dividend 3. Divisor 0. Quotient the fraction $3/0$. This fraction of which the denominator is cipher, is termed an infinite quantity. In this quantity consisting of that which has cipher for a divisor, there is no alteration, though many be inserted or extracted; as no change takes place in the infinite and immutable God."

He goes on, however, to note that $a_0 \cdot 0 = a$, so a full understanding had yet to be achieved.
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- How he achieved this feat, no one knows...
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- Power series expansions for \( \sin \) and \( \cos \)
- Series for \( \pi/4 \)
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