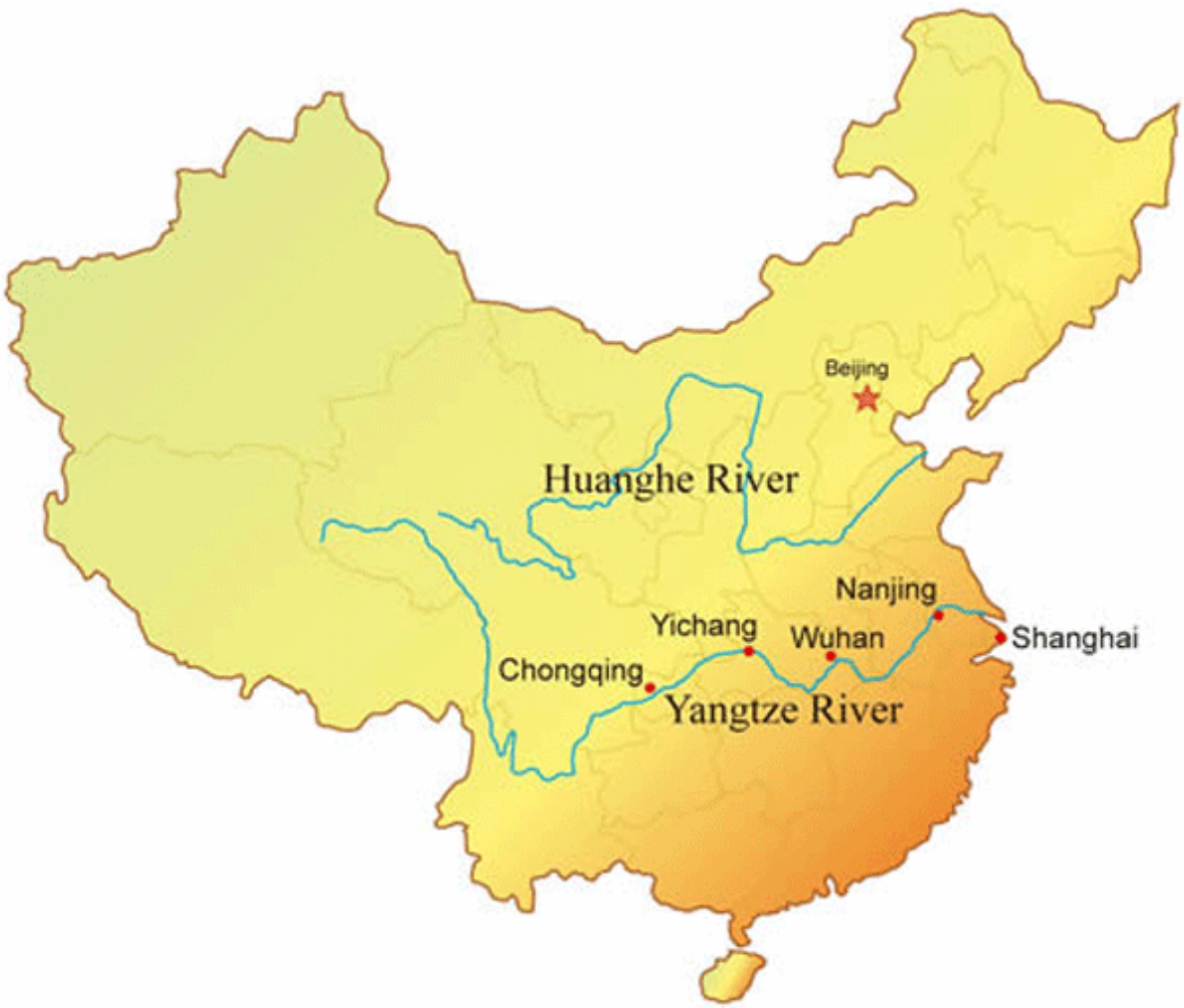
The Ancient and Medieval: China and India

Douglas Pfeffer

Table of contents

Outline


Beijing
Huanghe River
Nanjing
Yichang
Wuhan
Shanghai
Chongqing
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  - Unfortunately, however, far fewer mathematical texts have survived
  - The majority of what we have are not original texts, but replicas
    - The oldest text we currently have is the Zhoubi Suanjing (Chou Pei Suan Ching)

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  - Written in the form of a dialog between a prince and his minister regarding the calendar

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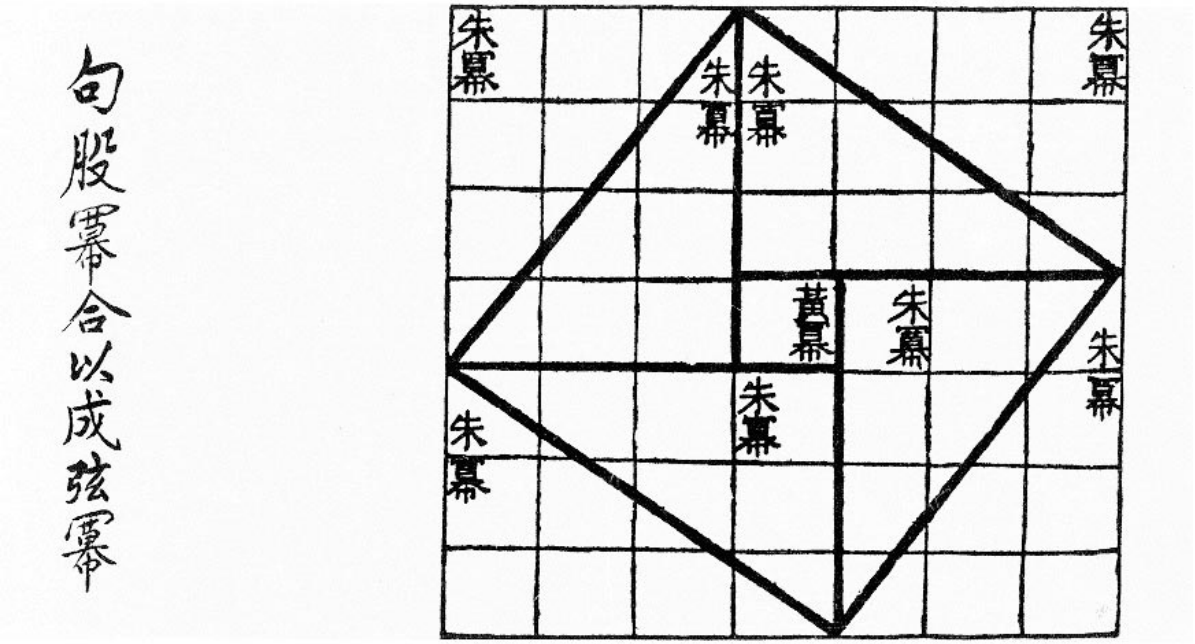

Its demonstration of the Pythagorean theorem

The Nine Chapters

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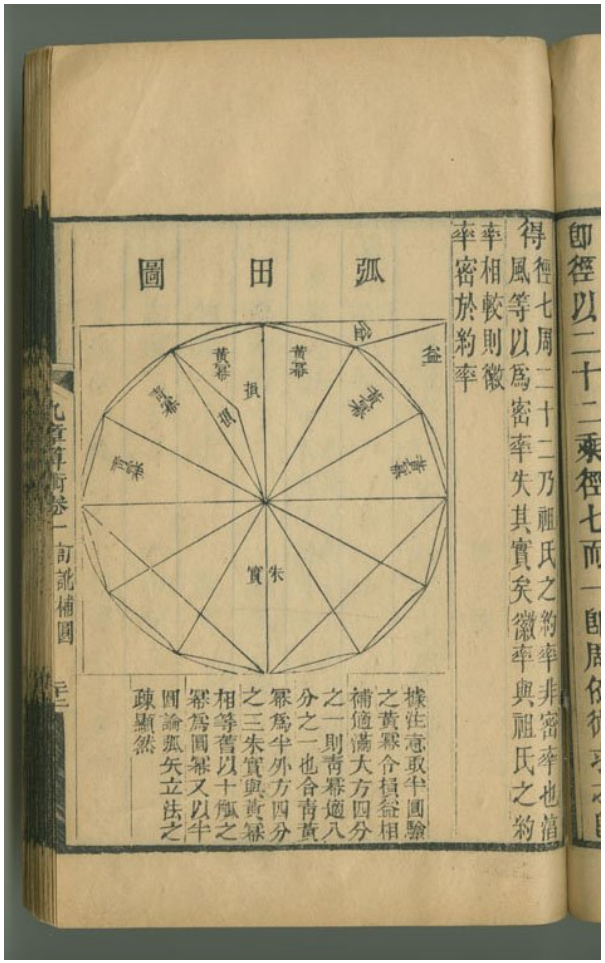

Jiuzhang Suanshu (or *Nine Chapters on the Mathematical Arts*)

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    - The final problem was one considering 4 equations and 5 unknowns

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- Fun fact: This is the smallest (and unique up to rotation and reflection) non-trivial case of a magic square, measuring 3×3

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It started by drafting up the following grid $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 1 & 1 \\ 26 & 34 & 39 \end{bmatrix}$

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- Through tedious descriptions:


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者三物者置程三物者皆列為行設其
方程之法以物數為率以直除之又列中行如右行數故物數
之禾以法之數故物數
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- They finished the problem by back solving the equations $36z = 99$, $5y + z = 24$, and $3x + 2y + z = 39$.

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  - The first (and less popular) was 'multiplicative': It had ciphers for $1 - 10$ and then for powers of 10.
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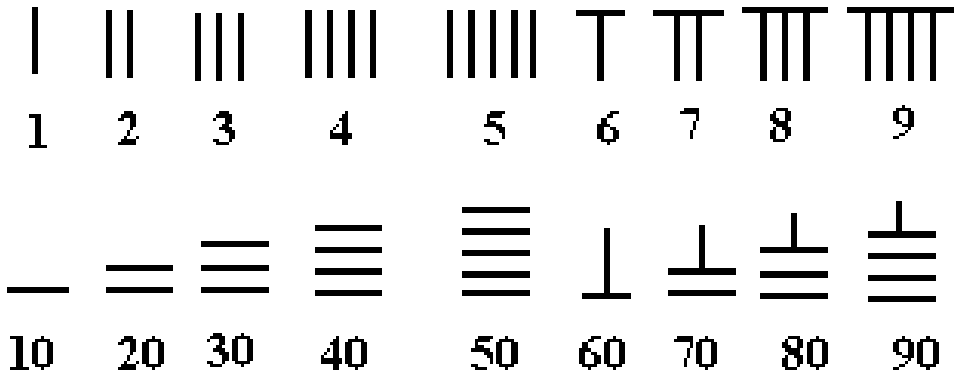
  - The second (and more popular) was the so-called Rod Numerals
    - This system was positional and had ciphers for $1 - 9$ and multiples of 10 up to 90
    - Numbers, as in the first system, were read in pairs.

Rod Numerals



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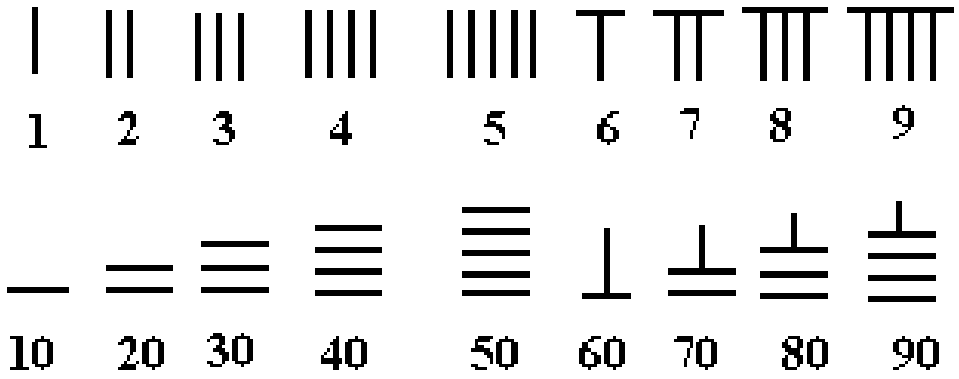

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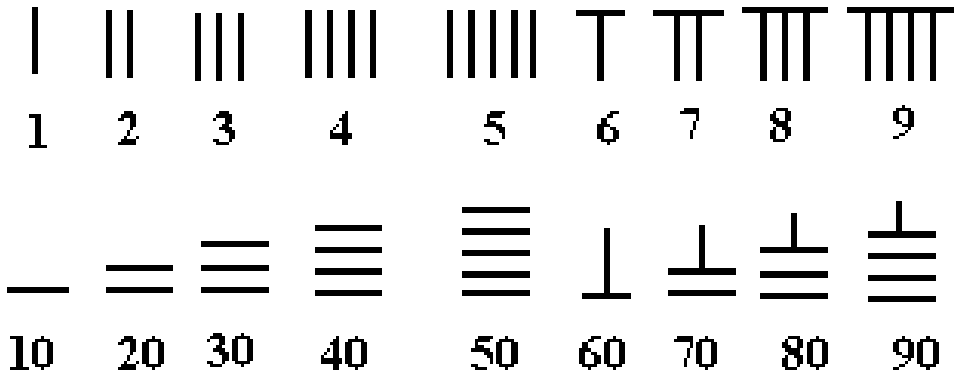

- Ex: $56,789$
- As in Babylonia, the 'empty position' came later with a round **O** symbol
  - In a 1247 CE text the value $1,405,536$ is given:
- This was the most common system and, since it was centesimal, was useful for computation.

The Abacus

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- Rod numerals weren't just a notation for computation: Actual bamboo rods were carried about in a bag by administrators and used as calculation devices on 'counting boards'
  - So dexterous were these counters that an 11th century writer described them as "flying so quickly that the eye could not follow their movement."

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  - So dexterous were these counters that an 11th century writer described them as "flying so quickly that the eye could not follow their movement."
- These counting boards anticipated the abacus
  - The modern abacus are relatively new (c. 1500s), but the concept dates back to the 500s

Chinese Remainder Theorem

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- The earliest known statement of the theorem appears in the 3rd-century book *Sunzi Suanjing* by Sunzi:
  - "There are certain things whose number is unknown. If we count them by threes, we have two left over; by fives, we have three left over; and by sevens, two are left over. How many things are there?"

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- Sunzi's work contains neither a proof nor a full algorithm.
- Much later, algorithms would be developed by Indian mathematicians Aryabhata (6th century) and Brahmagupta (7th century), and in Fibonacci's *Liber Abaci* (1202)

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- Of interest is that Liu Hui also, much as the Babylonians had much earlier, correctly calculated the volume of the frustrum of a right-pyramid
  - Oddly enough, when he tackled the frustrum of a cone, he just used $\pi = 3$.

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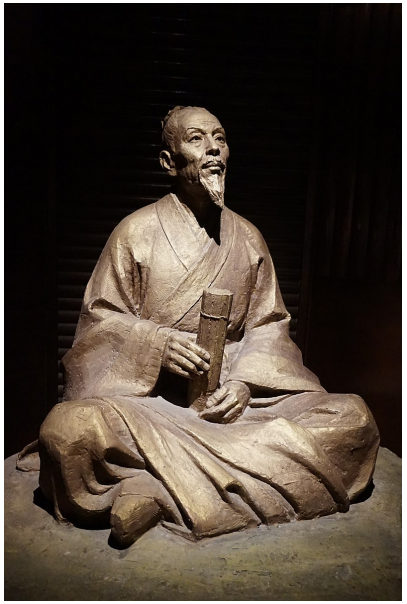
- The 400s saw the rise of mathematician Zu Chongzhi



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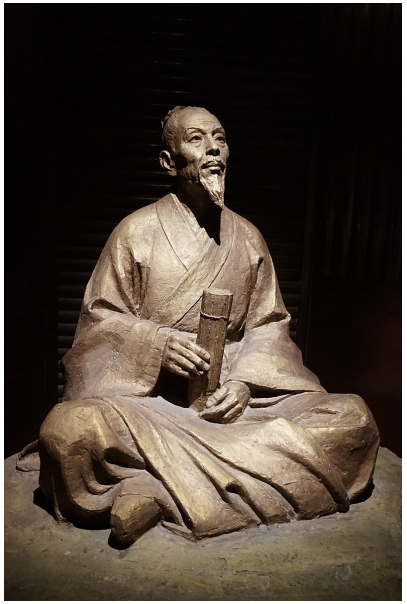

- He achieved: $\pi \approx \frac{355}{113} \approx \mathbf{3.14159}92$
  - This approximation was not matched until the 1400s
  - How Changzhi achieved this approximation is not known

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- From the 10th to the 13th centuries, no new mathematical breakthroughs seem to have occurred
  - This is interesting since these centuries saw the invention of paper, the compass, and gunpowder

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  - Contained 170 problems about circles inscribed within (or circumscribed without) a right triangle and determining the relationships between the sides and the radii

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- Of interest, however, is that these last two contributions are seldom attributed to Hui. Similar results were published in the more popular text *Precious Mirror* by Zhu Shiji

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  - In 1303, he wrote *Siyuan yujian* (*Jade Mirror of the Four Origins*)
    - The four origins were heaven, earth, man, and matter. They represented the four unknown quantities in a given equation.

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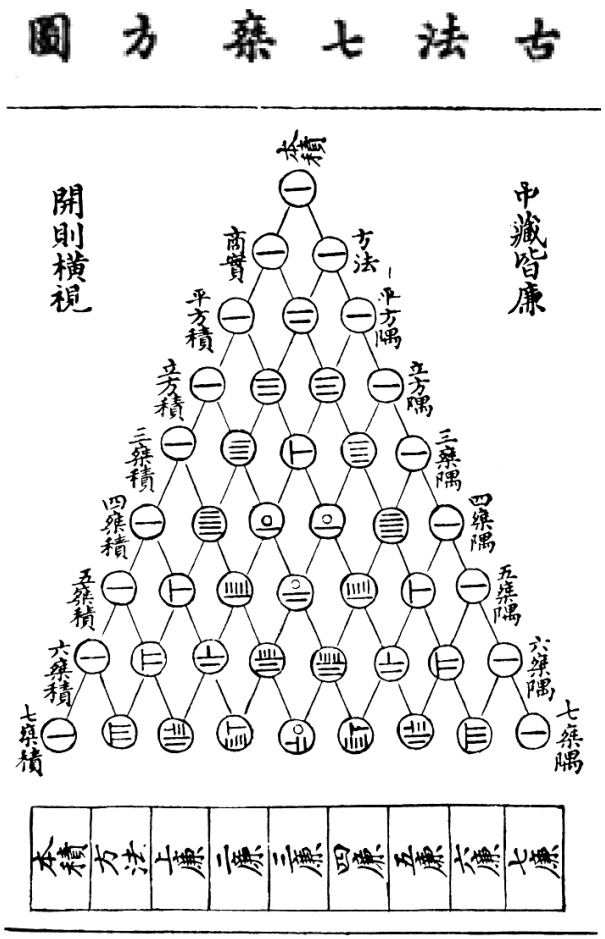
- Also included a systematic study of binomial coefficients and exhibited what is now mistakenly attributed to Blaise Pascal (1600s)
  - Featured Rod Numerals with the empty position **O**
  - Zhu himself claims the triangle is 'known' and 'old' and, notably, not his discovery

Chinese 'Pascals Triangle'



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- We now turn our eyes westward toward the Indian subcontinent...

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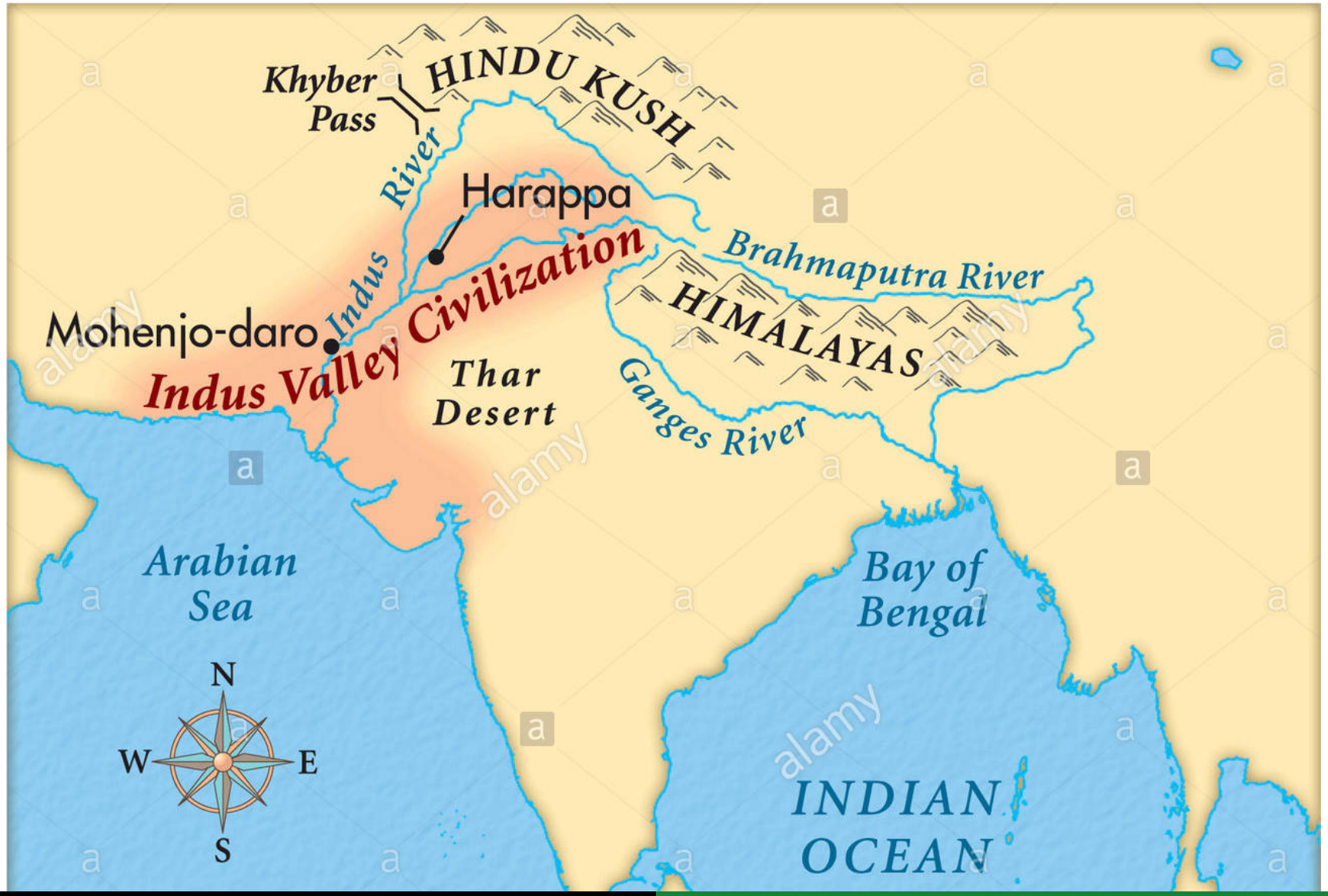
Khyber Pass
HINDU KUSH
Indus River
Harappa
Mohenjo-daro
Indus Valley Civilization
Brahmaputra River
HIMALAYAS
Thar Desert
Ganges River
Arabian Sea
Bay of Bengal
INDIAN OCEAN
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- The area, however, was volatile through movement and conquest. Even Indian languages were not entirely uniform.
- The Vedas, a group of ancient religious texts, do given detailed building prescriptions for altars and the like
  - These prescriptions came in the form of the *Sulbasutras* or "rules of the chord"
  - Eerily similar to Egyptian geometry (though few scholars support a strong connection to Egypt due to a lack of continuity in Indian mathematics)

Sulbasutras

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  - Contains, in part, of Pythagorean triples (although there is little concrete evidence of Mesopotamian influence)
- One such author, Apastamba, gave a number of geometric arguments
  - For example, a construction on how to, given a rectangle, draft up a square with the same area

Aryabhata

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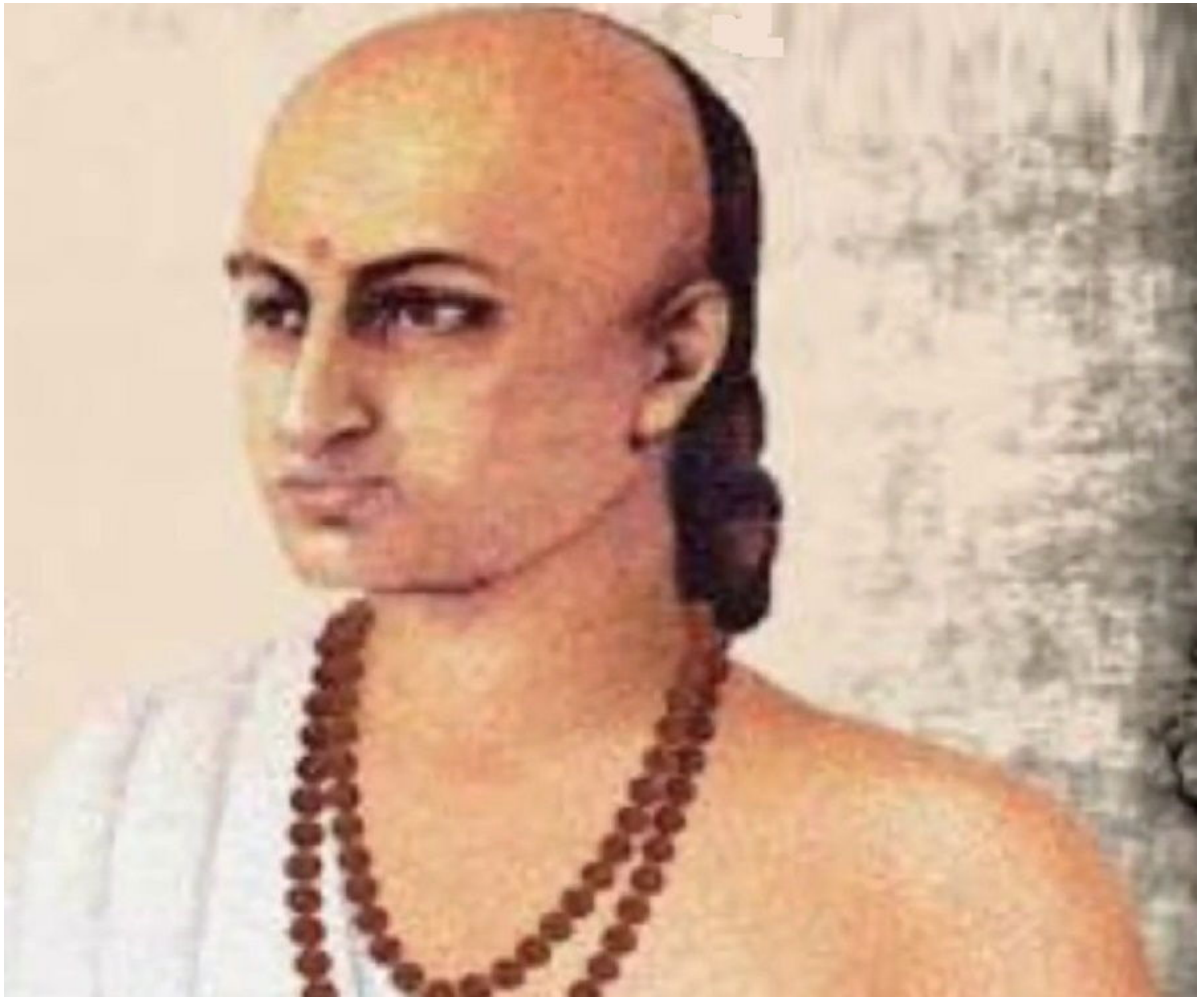

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  - Although the names of Indian mathematicians exist before him, none of their work survived
    - *Aryabhatiya*, much like Euclid's *Elemets*, was a summary of earlier developments compiled by a single author
    - Unlike *Elements*, however, *Aryabhatiya* exhibited no deductive methodology

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  - area of a trapezoid vs. area of an arbitrary plane figure

Aryabhatiya

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- By this estimate, $\pi \approx 3.1416$
  - Some scholars use this to argue the successes found within the *Aryabhatiya*

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    - The fact that Aryabhata may have been influenced by the Greeks is further supported by his adoption of the myriad as a unit length in geometry

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- Here,
  - $a =$ 'measure',
  - $b =$ 'fruit',
  - $c =$ 'desire', and
  - $x =$ 'fruit of the desire'

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  - This method was hypothesized to be formulated c. 12th century in India and then disseminated to China and Arabia and from Arabia to Italy in the 14th and 15th centuries

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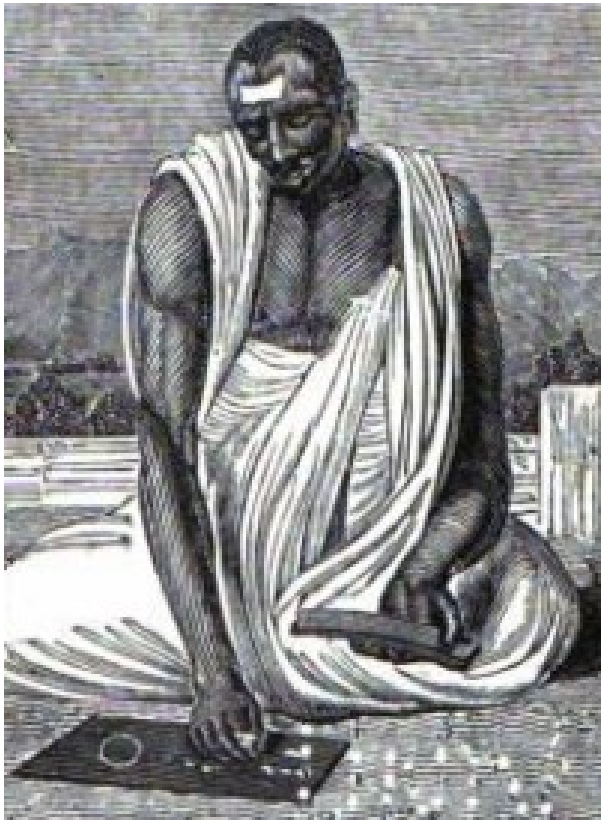

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  - Best known work was *Brahmasphuta Siddhanta*
    - Much like *Aryabhatiya*, it contained a mixture of correct and incorrect results

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  - Greeks knew how to subtract via geometric magnitudes, but Brahmagupta formalized these into numerical rules for positive and negative numbers
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- On the matter of zero, Brahmagupta seemed to have made a false start:
  - He argued that $\frac{0}{0} = 0$, but on $\frac{a}{0}$ for $a \neq 0$, he avoided it...
  - "Positive divided by positive, or negative by negative, is affirmative. Cipher divided by cipher is naught. Positive divided by negative is negative. Negative divided by affirmative is negative. Positive or negative divided by cipher is a fraction with that for denominator"

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  - Special cases of Pells equation was solved by the next prominent Indian mathematician Bhaskara in c. 1100

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He goes on, however, to note that $\frac{a}{0} \cdot 0 = a$, so a full understanding had yet to be achieved.

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- With the dawn on the Renaissance, however, the center for mathematical discoveries again began to find its way westward...