Lecture 22: Section 3.3
Properties of Logarithms

Properties:

$$\log_a(uv) = \log_a u + \log_a v$$

$$\log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$$

$$\log_a u^n = n \log_a u$$

Change of base formula

Recall the following properties of Logarithm:

The logarithmic function with base a

$y = f(x) = \log_a x$ if and only if

1. Domain of f:
2. $\log_a 1 =$
3. $\log_a a =$
4. $\log_a a^x =$ for all real number x

   $a^{\log_a x} =$ for $x > 0$

The Natural Logarithmic Function

$y = \ln x$ if and only if

Note the following:

$\ln 1 =$ $\ln e =$

$e^{\ln x} =$ $\ln(e^x) =$

Properties of Logarithms

Let u, v and a be positive real numbers with $a \neq 1$ and n be any real number. The following properties hold:

1. $\log_a(uv) =$

2. $\log_a\left(\frac{u}{v}\right) =$

3. $\log_a u^n =$

Proof:

NOTE: $\log_a(u+v) \neq \log_a u + \log_a v$

$(\log_a u)^n \neq n \log_a u$

ex. Evaluate:

1) $\log_4 2 + \log_4 32$

2) $\log_2 80 - \log_2 5$

3) $-\frac{1}{3}\log_4 8$

ex. Rewrite and simplify if possible:

1) $\ln(2 + e^x)$

2) $\log_2(x - y)$

3) $\frac{\log_3 x}{\log_3 y},\ y \neq 1$

4) $\ln\left(\frac{1}{\sqrt[3]{e}}\right)$

5) $\log_9\left(\frac{\sqrt[4]{9}}{3}\right)$

6) $2^{4\log_2 x}$

7) $\ln\sqrt{\frac{x^3y}{z}}$

ex. Rewrite and simplify:

1) $\ln \dfrac{\sqrt{x^3 e^{x-1}}}{x^2+1}$

2) $\log \sqrt{x\sqrt{y\sqrt{z}}}$

ex. Write as a single logarithm:

1) $\frac{1}{2}[\ln(x-5)+\ln x]-\ln(2y)$

2) $\log 2x-\log(x+1)-\frac{1}{3}\log(3x+7)$

Change of Base Formula

Let $a \neq 1,\ b \neq 1$ and x be positive real numbers. Then

$$\log_a x = \frac{\log_b x}{\log_b a}$$

When using a calculator, we need the specific formulas:

$$\log_a x = \frac{\log x}{\log a} \quad \text{or} \quad \log_a x = \frac{\ln x}{\ln a}$$

NOTE:

Most calculators have both 'log' and 'ln' keys to calculate the common and natural logarithm of a number. By using the Change of Base Formula, we can

1. evaluate logarithms to other bases.

2. graph logarithms to other bases.

ex. Given $\log 5 \approx 0.7, \log 3 \approx 0.48,\ \ln 3 \approx 1.1$, and $\ln(4+e) = 1.9$, use Change of Base Formula to find:

1) $\log_3 5$

2) $\log_{\sqrt{3}} \sqrt{4+e}$

ex. Solve $\log_3 x = \log_9(2x-1)$

Practice.

1) Write $\log_4 x + 4\log_2 y$ as a single logarithm with base 2.

2) Solve: $2\log_3 x = \log_9 16$

3) Solve: $\log_2 x = \log_4 25$

4) Solve: $\log_5 x = \log_{\sqrt{5}} 6$

Answer. 1) $\log_2 \sqrt{xy^4}$ 2) $x = 2$ 3) $x = 5$ 4) $x = 36$