MTG 7397 Due: April 23rd 12:50 NAME:

FINAL

1. Suppose that a chain map $f : C \to C'$ induces an isomorphism $[D, C] \to [D, C']$ for any chain complex D. Prove that f is a homotopy equivalence of chain complexes.

2. Let $G = \mathbb{Z}_n (= \mathbb{Z}/n\mathbb{Z})$. Show that every $\mathbb{Q}G$-module is projective.

3. Let P and M be $\mathbb{Z}G$-modules where P is projective and M is a free $\mathbb{Z}$-module. Show that $P \otimes M$ is a projective $\mathbb{Z}G$-module (for the diagonal action).

4. Let $0 \to A \to E \xrightarrow{\phi} G \to 1$ be a group extension corresponding to an element $\alpha \in H^2(G, A)$ where A is a G-module. Show that $\phi^*(\alpha) = 0$.

5. Let $cd(G) = n < \infty$ and $(G : H) < \infty$. Show that the transfer

$$tr : H^n(H, M) \to H^n(G, M)$$

is an epimorphism for any $\mathbb{Z}G$-module M.

6. Suppose that $K(\Gamma, 1)$ is finitely dominated by means of a homotopy retraction $r : Y \to K(\Gamma, 1)$ that induces isomorphism of the fundamental groups. Show that
(a) $cd(\Gamma) < \infty$.
(b) $\Gamma \in FP$.

7. Let $\phi : G \to H$ be a surjective homomorphism. Show that there is an H-module M such that the induced homomorphism

$$\phi^* : H^1(H, M) \to H^1(G, M)$$

is nonzero.

8.(Extra credit) Prove that $H_2(G) = 0$ for any finite subgroup $G \subset S^3$.