FINAL

Due: 12:50 PM, April 24rd 2024

1. Show that every map $f : S^{k+l} \to S^k \times S^l$, $k, l > 0$, induces the trivial homomorphism of reduced cohomology groups $f^* : \tilde{H}^*(S^k \times S^l) \to \tilde{H}^*(S^{k+l})$.

2. Show that there is no map $f : \mathbb{C}P^m \to \mathbb{C}P^n$ with $m > n$ inducing a nontrivial homomorphism $f^* : H^2(\mathbb{C}P^n) \to H^2(\mathbb{C}P^m)$.

3. Show that RP^{2n+1} is not homotopy equivalent to $RP^{2n} \vee S^{2n+1}$

4. Let $U \subset \mathbb{R}^n$ be an open set. Show that the groups $H_i(U)$ are countable for all i.

5. Show that a compact manifold does not retract onto its boundary.

6. Show that $H_c^{n+1}(X \times \mathbb{R}) = H_c^n(X)$ for all n.

7. Show that the tensor product commutes with direct limits:

$$(\lim_{\to} G_\alpha) \otimes A = \lim_{\to} (G_\alpha \otimes A).$$

8. Show that for orientable surfaces M_g and M_h of genus g and h with $g > h$

(a) there is no map $f : M_h \to M_g$ of degree one;

(b) there is no retraction of M_{g+h} onto $M_g \setminus IntD^2$ where $M_{g+h} = M_g \# M_h$ be the connected sum, i.e.,

$$M_{g+h} = (M_g \setminus IntD^2) \cup_{\partial D^2} (M_h \setminus IntD^2).$$