Show that the set $B(\mathbb{R}, \mathbb{R})$ is closed in $\mathbb{R}^R$ in the uniform topology, but not in the topology of compact convergence.

**SOLUTION.** Let $f_n \to f$ be a sequence of bounded functions that converge uniformly to a function $f : \mathbb{R} \to \mathbb{R}$. We show that $f$ is bounded. By the definition of uniform convergence applied with $\epsilon = 1/2$, there is $N$ such that $|f(x) - f_N(x)| < 1/2$ for all $x \in \mathbb{R}$. Let $M = \sup f_N$. Then $\sup f \leq M + 1/2$. Indeed, by the triangle inequality, $|f(x)| \leq |f_N(x)| + |f(x) - f_N(x)| \leq M + 1/2$.

The sequence of functions defined by the formula $f_n(x) = \min\{x^2, n\}$ converges to $f(x) = x^2$ in the topology of compact convergence. Indeed, it convergence to $f$ on every interval $[-m, m]$ and hence, on every compact set $C \subset \mathbb{R}$. 
