Topology, MTG 4930/6346 Fall-23 SOLUTIONS

Quiz 3

1. Compute the simplicial homology groups of the triangular parachute obtained from Δ^2 by identifying its three vertices to a single point.

Everbody solved this one!

2. Compute $H_n(X, A)$ where $X = T\#T$ is a connected sum of two tori (a closed orientable surface of genus 2) and A is the separating circle.

SOLUTION. Since (X, A) is a good pair, $H_n(X, A) = \tilde{H}(X/A)$. Note that $X/A = T \vee T$ where T is a torus. Thus, $H_n(X, A) = \tilde{H}_n(T \vee T) = \tilde{H}_n(T) \oplus \tilde{H}_n(T)$. Since homology groups of torus were computed in class, $\tilde{H}_0(T) = 0$, $\tilde{H}_1(T) = \mathbb{Z}\oplus\mathbb{Z}$, $\tilde{H}_2(T) = \mathbb{Z}$, and $\tilde{H}_n(T) = 0$ for $n > 2$, we obtain that $H_0(X, A) = 0$, $H_1(X, A) = \oplus^4\mathbb{Z}$, $H_2(X, A) = \mathbb{Z} \oplus \mathbb{Z}$, and $H_n(X, A) = 0$ for $n > 2$.

3. (extra credit) Does there exist a short exact sequence

$$0 \to \mathbb{Z}_4 \to \mathbb{Z}_8 \oplus \mathbb{Z}_2 \to \mathbb{Z}_4 \to 0?$$

SOLUTION. Yes, if we take the monomorphism $\alpha : \mathbb{Z}_4 \to \mathbb{Z}_8 \oplus \mathbb{Z}_2$ defined as $\alpha(1) = (2, 1)$, then the quotient group $(\mathbb{Z}_8 \oplus \mathbb{Z}_2)/Im(\alpha)$ is isomorphic to $\mathbb{Z}_4$. Indeed, the element $(1, 1) + Im(\alpha)$ has order 4: $2(1, 1) \notin Im(\alpha)$ as well as $3(1, 1) \notin Im(\alpha)$ and, clearly, $4(1, 1) \in Im(\alpha)$.