MAP 2302  Spring-2016  Quiz 1

1. Find values of $m$ such that the function $y = e^{mx}$ is a solution to the equation $y'' - 5y' + 6y = 0$.

SOLUTION: Since $y' = me^{mx}$ and $y'' = m^2e^{mx}$, we obtain

$$m^2e^{mx} - 5me^{mx} + 6e^{mx} = 0 \iff e^{mx}(m^2 - 5m + 6) = 0.$$ 

Since $e^{mx} \neq 0$, this equation is equivalent to $m^2 - 5m + 6 = 0$. Note that $m^2 - 5m + 6 = (m - 2)(m - 3)$. Hence the roots are $m = 2$ and $m = 3$. Thus, $y = e^{2x}$ and $y = 3x$ are the solutions to $y'' - 5y' + 6y = 0$.

2. Solve the IVP $y'' - y = 0, \quad y(0) = 1, \quad y'(0) = 2$.

SOLUTION: Since the linear combination of $e^x$ and $e^{-x}$ is a solution to $y'' - y = 0$, we consider the system of equations $y(0) = 1, \, y'(0) = 2$ for $y = c_1e^x + c_2e^{-x}$. Note that $y' = c_1e^x - c_2e^{-x}$.

Therefore, $c_1 + c_2 = 1$ and $c_1 - c_2 = 2$. Add these two equations to obtain $2c_1 = 3$ and hence $c_1 = \frac{3}{2}$. From the 1st equation we find that $c_2 = 1 - \frac{3}{2} = -\frac{1}{2}$.

Thus, $y = \frac{3}{2}e^x - \frac{1}{2}e^{-x}$ is the solution to the IVP.