MAC 2313
Problem Set #1

0. Do all WebAssign problems for Lectures 1, 2, 3.

1. Find a unit vector in the direction of $\langle 1, 2, 3 \rangle$.

2. Let $P = (a, 1, -1), Q = (0, 1, 1)$ and $R = (a, -1, 3)$. For what values of $a$ is $\triangle PQR$ a right triangle?

3. Let $\vec{A} = \langle 3, 5, 7 \rangle, \vec{B} = \langle 3, 4, 0 \rangle$. Find the component of $\vec{A}$ in the direction of $\vec{B}$.

4. Find the area of the parallelogram with vertices $A(-3, 0), B(-1, 6), C(8, 5)$ and $D(6, -1)$ first using the determinant interpretation, and then using cross products.

5. Let $P$ be a point not on the line $L$ that passes through the points $Q$ and $R$. Show that the distance $d$ from the point $P$ to the line $L$ is
   \[ d = \frac{\| \vec{a} \times \vec{b} \|}{\| \vec{a} \|} \]
   where $\vec{a} = \vec{QR}, \vec{b} = \vec{QP}$.

6. Let $P$ be a point not on the plane that passes through the points $Q, R$ and $S$. Show that the distance $d$ from $P$ to the plane is
   \[ d = \frac{\| \vec{a} \cdot (\vec{b} \times \vec{c}) \|}{\| \vec{a} \times \vec{b} \|} \]
   where $\vec{a} = \vec{QR}, \vec{b} = \vec{QS}$ and $\vec{c} = \vec{QP}$.

7. Prove that the altitudes of a triangle intersect at a point.

8. Prove that the medians of a triangle intersect at a point.

9. Let $ABCD$ be a quadrilateral. Let $P, Q, R, S$ be the midpoints of $AB, BC, CD, DA$, respectively. Prove that $PQRS$ is a parallelogram.

10. Prove that the diagonals of a parallelogram are perpendicular if and only if the parallelogram is a rhombus (i.e. its four sides have equal length).

11. Try making up problems that you can solve, but you think other students would have a hard time figuring out. This forces you to think about concepts very deeply, and in turn you learn a lot.