Name ______________

Quiz 5

Put a box around your answer and show all work!

1. Using the limit definition, find the derivative of $f(x) = 3x^2 - 5$. We use the limit definition to find the derivative of f.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{(3(x+h)^2 - 5) - (3x^2 - 5)}{h} \\
&= \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 - 5 - 3x^2 + 5}{h} \\
&= \lim_{h\to 0} \frac{6xh + 3h^2}{h} \\
&= \lim_{h\to 0} (6x + 3h) \\
&= \boxed{6x}.
\end{aligned}$$

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2. (Bonus) Evaluate

$$\lim_{x\to -\infty} \left(x + \sqrt{x^2 + 4x + 5} \right).$$

Note that substitution gives $-\infty + \infty$, so we have to try harder. We first multiply by a form of 1 involving the conjugate of $x + \sqrt{x^2 + 4x + 5}$:

$$\begin{aligned}
x + \sqrt{x^2 + 4x + 5} &= \left(x + \sqrt{x^2 + 4x + 5} \right) \cdot \frac{x - \sqrt{x^2 + 4x + 5}}{x - \sqrt{x^2 + 4x + 5}} \\
&= \frac{x^2 - (x^2 + 4x + 5)}{x - \sqrt{x^2 + 4x + 5}} \\
&= \frac{-4x - 5}{x - \sqrt{x^2 + 4x + 5}}
\end{aligned}$$

Now, to compute our limit, we divide both the numerator and denominator by a form of x. Note that, since the values of x we care about are negative, we have $x = -\sqrt{x^2}$.

$$\begin{aligned}
\lim_{x\to-\infty} x + \sqrt{x^2+4x+5} &= \lim_{x\to-\infty} \frac{-4x-5}{x-\sqrt{x^2+4x+5}} \\
&= \lim_{x\to-\infty} \frac{-4x-5}{x-\sqrt{x^2+4x+5}} \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)} \\
&= \lim_{x\to-\infty} \frac{-4-\frac{5}{x}}{1-\frac{1}{(-\sqrt{x^2})}\sqrt{x^2+4x+5}} \\
&= \lim_{x\to-\infty} \frac{-4-\frac{5}{x}}{1+\sqrt{1+\frac{4}{x}+\frac{5}{x^2}}} \\
&= \frac{-4}{1+\sqrt{1}} = \boxed{-2}.
\end{aligned}$$

3. Find the derivatives of the given functions:

(a) $f(x) = \dfrac{x^2-4}{x-2}$

We factor the numerator, cancel terms, and take the derivative of what remains:

$$\begin{aligned}
\frac{d}{dx} f(x) &= \frac{d}{dx} \frac{x^2-4}{x-2} \\
&= \frac{d}{dx} \frac{(x-2)(x+2)}{x-2} \\
&= \frac{d}{dx}(x+2) \\
&= \boxed{1}.
\end{aligned}$$

(b) $f(x) = e^x \cdot x^3$

We use the product rule:

$$\begin{aligned}
\frac{d}{dx}\,f(x) &= \frac{d}{dx}\,e^x \cdot x^3 \\
&= e^x\,\frac{d}{dx}(x^3) + \frac{d}{dx}(e^x)x^3 \\
&= e^x \cdot 3x^2 + e^x \cdot x^3 \\
&= \boxed{e^x \cdot x^2(3+x)}.
\end{aligned}$$

(c) $f(x) = x^{\sin(\pi/2)}$

Note that $\sin(\pi/2) = 1$.

$$\begin{aligned}
\frac{d}{dx}\,f(x) &= \frac{d}{dx}\,x^1 \\
&= \boxed{1}.
\end{aligned}$$

(d) $f(x) = -\dfrac{x}{e^x}$

We use the quotient rule:

$$\begin{aligned}
\frac{d}{dx}\,f(x) &= -\,\frac{d}{dx}\,\frac{x}{e^x} \\
&= -\frac{e^x\,\frac{d}{dx}(x) - x\,\frac{d}{dx}(e^x)}{(e^x)^2} \\
&= -\frac{e^x - x \cdot e^x}{(e^x)^2} \\
&= -\frac{e^x(1-x)}{(e^x)^2} \\
&= -\frac{1-x}{e^x} \\
&= \boxed{\frac{x-1}{e^x}}.
\end{aligned}$$

Name ______________

Quiz 5

Put a box around your answer and show all work!

1. Using the limit definition, find the derivative of $f(x) = 5x^2+3$. We use the limit definition to find the derivative of f.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{(5(x+h)^2+3)-(5x^2+3)}{h} \\ &= \lim_{h\to 0} \frac{5x^2+10xh+5h^2+3-5x^2-3}{h} \\ &= \lim_{h\to 0} \frac{10xh+5h^2}{h} \\ &= \lim_{h\to 0}(10x+5h) \\ &= \boxed{10x}. \end{aligned}$$

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2. (Bonus) Evaluate

$$\lim_{x\to -\infty} \left(x+\sqrt{x^2+4x+5}\right).$$

Note that substitution gives $-\infty+\infty$, so we have to try harder. We first multiply by a form of 1 involving the conjugate of $x+\sqrt{x^2+4x+5}$:

$$\begin{aligned} x+\sqrt{x^2+4x+5} &= \left(x+\sqrt{x^2+4x+5}\right)\cdot\frac{x-\sqrt{x^2+4x+5}}{x-\sqrt{x^2+4x+5}} \\ &= \frac{x^2-(x^2+4x+5)}{x-\sqrt{x^2+4x+5}} \\ &= \frac{-4x-5}{x-\sqrt{x^2+4x+5}} \end{aligned}$$

Now, to compute our limit, we divide both the numerator and denominator by a form of x. Note that, since the values of x we care about are negative, we have $x = -\sqrt{x^2}$.

$$\begin{aligned}
\lim_{x\to-\infty} x + \sqrt{x^2+4x+5} &= \lim_{x\to-\infty} \frac{-4x-5}{x-\sqrt{x^2+4x+5}} \\
&= \lim_{x\to-\infty} \frac{-4x-5}{x-\sqrt{x^2+4x+5}} \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)} \\
&= \lim_{x\to-\infty} \frac{-4-\frac{5}{x}}{1-\frac{1}{(-\sqrt{x^2})}\sqrt{x^2+4x+5}} \\
&= \lim_{x\to-\infty} \frac{-4-\frac{5}{x}}{1+\sqrt{1+\frac{4}{x}+\frac{5}{x^2}}} \\
&= \frac{-4}{1+\sqrt{1}} = \boxed{-2}.
\end{aligned}$$

3. Find the derivatives of the given functions:

(a) $f(x) = \dfrac{e^x}{x}$

We use the quotient rule:

$$\begin{aligned}
\frac{d}{dx} f(x) &= \frac{d}{dx} \frac{e^x}{x} \\
&= \frac{x\,\frac{d}{dx}(e^x) - e^x\,\frac{d}{dx}(x)}{x^2} \\
&= \frac{x \cdot e^x - e^x}{x^2} \\
&= \boxed{\frac{(x-1)e^x}{x^2}}.
\end{aligned}$$

(b) $f(x) = x^{\cos(\pi)}$

Note that $\cos(\pi) = -1$.

$$\frac{d}{dx} f(x) = \frac{d}{dx} x^{-1}$$
$$= \boxed{-x^{-2}}.$$

(c) $f(x) = x^2 \cdot e^x$

We use the product rule:

$$\frac{d}{dx} f(x) = x^2 \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(x^2)$$
$$= x^2 \cdot e^x + e^x \cdot 2x$$
$$= \boxed{e^x \cdot x(x+2)}.$$

(d) $f(x) = \dfrac{x^2 - 9}{x+3}$

We factor the numerator, cancel terms, and take the derivative of what remains.

$$\frac{d}{dx} f(x) = \frac{d}{dx} \frac{x^2 - 9}{x+3}$$
$$= \frac{d}{dx} \frac{(x-3)(x+3)}{x+3}$$
$$= \frac{d}{dx}(x-3)$$
$$= \boxed{1}.$$