Name Solution

Quiz 1

1. Let $f(x) = \sqrt{4-x^2}$ and $g(x) = \dfrac{1}{x^2-4}$. Find $(g \circ f)(x)$ and its domain.

Here,

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= \frac{1}{(\sqrt{4-x^2})^2 - 4} \\ &= \frac{1}{4-x^2-4} \\ &= -\frac{1}{x^2}.\end{aligned}$$

The domain of $(g \circ f)$ is the domain of $-\frac{1}{x^2}$ intersected with the domain of the 'inside' function f. The domain of $-\frac{1}{x^2}$ is wherever x^2 is not 0, which is everything but 0,

$$(-\infty, 0) \cup (0, \infty),$$

while the domain of f is wherever $4 - x^2$ is at least 0, which is,

$$[-2, 2].$$

Then intersection of these is

$$[-2, 0) \cup (0, 2].$$

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2. Solve for x:

$$\frac{1}{x+1} + \frac{3}{x-1} + 2 = 0.$$

First, we get a common denominator.

$$\frac{1}{x+1} + \frac{3}{x-1} + 2 = 0 \implies$$

$$\frac{1}{x+1}\frac{(x-1)}{(x-1)} + \frac{3}{x-1}\frac{(x+1)}{(x+1)} + 2\frac{(x-1)(x+1)}{(x-1)(x+1)} = 0 \implies$$

$$\frac{(x-1) + 3(x+1) + 2(x-1)(x+1)}{(x-1)(x+1)} = 0 \implies$$

$$\frac{2x^2 + 4x}{x^2 - 1} = 0.$$

Now, multiplying both sides by the denominator of the lefthand side, we have

$$2x^2 + 4x = 2x(x+2) = 0,$$

and this gives solutions when $x = 0$ or $x + 2 = 0$, so either $x = 0$ or $x = -2$.

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3. Find all values of θ in $[0, 2\pi)$ whose reference angle is $\pi/6$.

   Note that the reference angle of θ is the angle in $[0, \pi/2)$ which θ makes with the x-axis. Thus, all angles θ in $[0, 2\pi)$ whose reference angle is $\pi/6$ are $\pi/6$, $5\pi/6$, $7\pi/6$, and $11\pi/6$.

Name ______________

Quiz 1

1. Let $f(x) = \dfrac{1}{x-2}$ and $g(x) = \dfrac{x}{x+1}$. Find $(g \circ f)(x)$ and its domain.

Here,

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= \frac{\left(\frac{1}{x-2}\right)}{\left(\frac{1}{x-2}\right) + 1}.\end{aligned}$$

Now, we multiply the numerator and the denominator by $(x-2)$ to simplify:

$$\begin{aligned}(g \circ f)(x) &= \frac{(x-2)\left(\frac{1}{x-2}\right)}{(x-2)\left(\frac{1}{x-2}\right) + (x-2)} \\ &= \frac{1}{1 + x - 2} \\ &= \frac{1}{x-1}.\end{aligned}$$

The domain of $(g \circ f)$ is the domain of $\frac{1}{x-1}$ intersected with the domain of the 'inside' function f. The domain of $\frac{1}{x-1}$ is wherever $x-1$ is not 0, which is everything but 1,

$$(-\infty, 1) \cup (1, \infty),$$

while the domain of f is wherever $x-2$ is not 0, which is everything but 2,

$$(-\infty, 2) \cup (2, \infty).$$

The intersection of these is everything but the points 1 and 2, given in interval notation as

$$(-\infty, 1) \cup (1, 2) \cup (2, \infty).$$

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2. Solve for x:
$$\frac{3}{x+1} + \frac{1}{x-1} = 2.$$

We first get a common denominator:

$$\begin{aligned} \frac{3}{x+1} + \frac{1}{x-1} &= 2 \implies \\ \frac{3}{x+1}\frac{(x-1)}{(x-1)} + \frac{1}{x-1}\frac{(x+1)}{(x+1)} &= 2 \implies \\ \frac{3(x-1)+(x+1)}{(x+1)(x-1)} &= 2. \end{aligned}$$

Now, we multiply both sides by the denominator on the left hand side,

$$\begin{aligned} 3x - 3 + x + 1 = 2(x+1)(x-1) &\implies \\ 4x - 2 = 2x^2 - 2 &\implies \\ 0 = 2x^2 - 4x &\implies \\ 0 = 2x(x-2). \end{aligned}$$

The solutions now are where either $x = 0$ or $x - 2 = 0$, meaning $x = 0$ or $x = 2$. These are our solutions.

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3. Find all values of θ in $[0, 2\pi)$ whose reference angle is $\pi/3$.

Note that the reference angle of θ is the angle it makes with the x-axis, so the angles θ in $[0, 2\pi)$ whose reference angle is $\pi/3$ are $\pi/3$, $2\pi/3$, $4\pi/3$, and $5\pi/3$.