Name Solution

Quiz 2

1. Let $f(x) = \dfrac{\sqrt{e^x - 1}}{x}$. Find the domain of f in interval notation.

   Since we have a square root involved, we must force the expression inside the square root $e^x - 1$ to be at least 0:

$$\begin{aligned} e^x - 1 \geq 0 &\implies \\ e^x \geq 1. \end{aligned}$$

   We may now take the natural logarithm of either side to get

$$\begin{aligned} e^x \geq 1 &\implies \\ x \geq \ln 1 &\implies \\ x \geq 0. \end{aligned}$$

   We must also assert that the denominator, x, is not 0, so our solution set is everything to the right of 0:

   $\boxed{(0, \infty)}$

2. Find the solution set:

$$2\log(x+1) - \log(2x+1) = 0$$

   First, we note that any solution we give must lie in the domain of the problem, $[-1/2, \infty)$. Now, we may move one term to the righthand side and use logarithm rules to simplify:

$$\begin{aligned} 2\log(x+1) - \log(2x+1) = 0 &\implies \\ 2\log(x+1) = \log(2x+1) &\implies \\ \log((x+1)^2) = \log(2x+1). \end{aligned}$$

   We may 'e' both sides to cancel logs, meaning

$$\begin{aligned} \log((x+1)^2) = \log(2x+1) &\implies \\ e^{\log((x+1)^2)} = e^{\log(2x+1)} &\implies \\ (x+1)^2 = (2x+1). \end{aligned}$$

Now, we've reduced this to a simple quadratic equation, which we can readily solve:

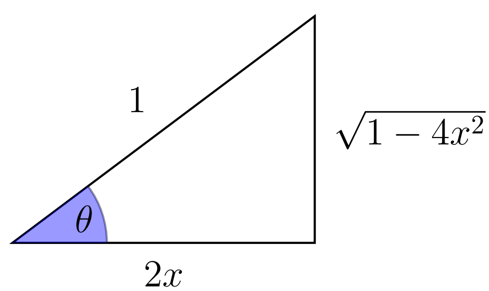
$$(x+1)^2 = (2x+1) \implies$$
$$x^2 + 2x + 1 = 2x + 1 \implies$$
$$x^2 = 0 \implies$$
$$x = 0.$$

This is in our domain, so we are done.

$\boxed{x = 0}$

3. Use a triangle to simplify the expression $\tan(\cos^{-1}(2x))$.

Let $\theta = \cos^{-1}(2x)$. Since $\cos(\theta) = 2x$, we may fill in a triangle to calculate $\tan(\theta)$.



Once we've filled in our triangle using $\cos(\theta) = 2x$ and the Pythagorean Theorem, we see that $\tan(\theta) = \frac{\sqrt{1-4x^2}}{2x}$.

$$\boxed{\frac{\sqrt{1-4x^2}}{2x}}$$

Name Solution

Quiz 2

1. Let $f(x) = \log(\sqrt{x^2 - 1})$. Find the domain of f in interval notation.
   Since a logarithm is involved, we must ensure its argument $\sqrt{x^2 - 1}$ is positive. A square root is positive exactly when its inside is positive, so we must have

$$\begin{aligned} x^2 - 1 > 0 &\implies \\ x^2 > 1 &\implies \\ |x| > 1, \end{aligned}$$

   which in interval notation is $(-\infty, -1) \cup (1, \infty)$. We must also check that the 'inside' of the square root, $x^2 - 1$ is at least 0, but since we already asserted $x^2 - 1$ must be positive, we are done.
   $\boxed{(-\infty, -1) \cup (1, \infty)}$

2. Find the solution set:

$$2\log(x) - \log(3x + 4) = 0$$

   We use the rules of logarithms to simplify:

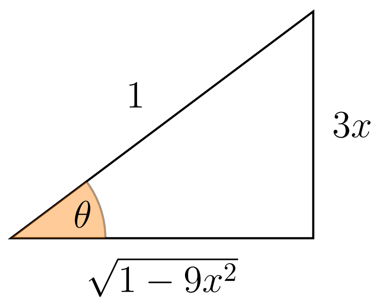
$$\begin{aligned} 2\log(x) - \log(3x + 4) = 0 &\implies \\ \log(x^2) - \log(3x + 4) = 0 &\implies \\ \log\left(\frac{x^2}{3x + 4}\right) = 0 &\implies \\ \frac{x^2}{3x + 4} = 10^0 &\implies \\ \frac{x^2}{3x + 4} = 1 &\implies \\ x^2 = 3x + 4 &\implies \\ x^2 - 3x - 4 = 0 &\implies \\ (x - 4)(x + 1) = 0. \end{aligned}$$

Now, x is either -1 or 4. The domain of our original expression is $(0, \infty)$, so 4 is our only solution.

$\boxed{x = 4}$

3. Use a triangle to simplify the expression $\cot(\sin^{-1}(3x))$.

Let $\theta = \sin^{-1}(3x)$. Since $\sin(\theta) = 3x$, we may fill in a triangle to calculate $\cot(\theta)$.



Once we've filled in our triangle using $\sin(\theta) = 3x$ and the Pythagorean Theorem, we see that $\cot(\theta) = \frac{\sqrt{1-9x^2}}{3x}$.

$$\boxed{\frac{\sqrt{1 - 9x^2}}{3x}}$$