Name Solution

Quiz 3

Put a box around your answer, please and thank you!

1. An escaping balloon has height $y(t) = 4t^2 + 4$ after t seconds have passed. Find a formula for the average velocity on $[1, 1+h]$.

   We use the formula for the average rate of change of a function over an interval.

$$\begin{aligned}\text{average velocity over } [1, 1+h] &= \frac{y(1+h) - y(1)}{(1+h) - 1} \\ &= \frac{(4(1+h)^2 + 4) - (4(1)^2 + 4)}{h} \\ &= \frac{4 + 8h + 4h^2 + 4 - (4+4)}{h} \\ &= \frac{8h + 4h^2}{h} \\ &= 8 + 4h.\end{aligned}$$

   This is our average velocity!

   $\boxed{8 + 4h}$

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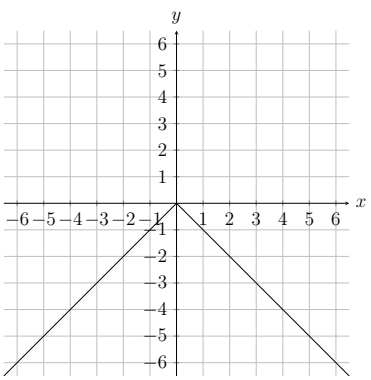
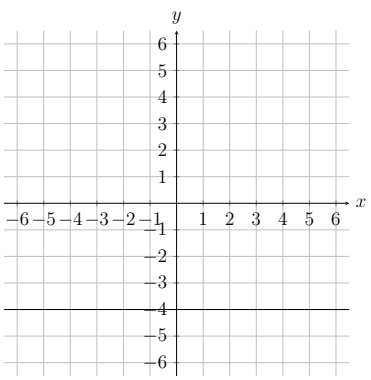
2. Sketch the graph of an *even* function f such that

$$\lim_{x \to 4} f(x) = -4.$$

   There are *many* good solutions here, but we give two easy ones:





3. Evaluate

$$\lim_{x\to 2} \frac{\frac{1}{x} - \frac{1}{2}}{2 - x}.$$

We simplify the given fraction and compute the remaining (simple) limit.

$$\begin{aligned}
\frac{\frac{1}{x} - \frac{1}{2}}{2 - x} &= \frac{\frac{1}{x} - \frac{1}{2}}{2 - x} \cdot \frac{2x}{2x} \\
&= \frac{2 - x}{(2 - x)(2x)} \\
&= \frac{1}{2x}.
\end{aligned}$$

Now, computing our limit is easy:

$$\begin{aligned}
\lim_{x\to 2} \frac{\frac{1}{x} - \frac{1}{2}}{2 - x} &= \lim_{x\to 2} \frac{1}{2x} \\
&= \frac{1}{2(2)} \\
&= \frac{1}{4}.
\end{aligned}$$

$\boxed{\frac{1}{4}}$

Name Solution

Quiz 3

Put a box around your answer, please and thank you!

1. An falling boulder has height $y(t) = -4t^2 - 4$ after t seconds have passed. Find a formula for the average velocity on $[1, 1+h]$.

   We use the formula for the average rate of change of a function over an interval.

$$\begin{aligned}\text{average velocity over } [1, 1+h] &= \frac{y(1+h) - y(1)}{(1+h) - 1} \\ &= \frac{(-4(1+h)^2 4) - (-4(1)^2 - 4)}{h} \\ &= \frac{-4 - 8h - 4h^2 - 4 - (-4 - 4)}{h} \\ &= \frac{-8h - 4h^2}{h} \\ &= -8 - 4h.\end{aligned}$$

   This is our average velocity!

   $\boxed{-8 - 4h}$

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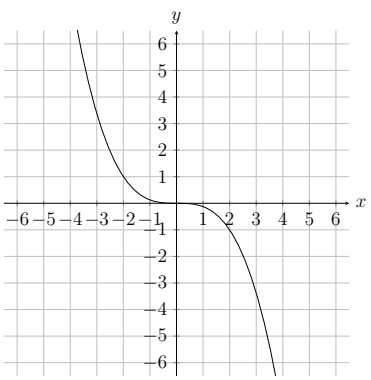
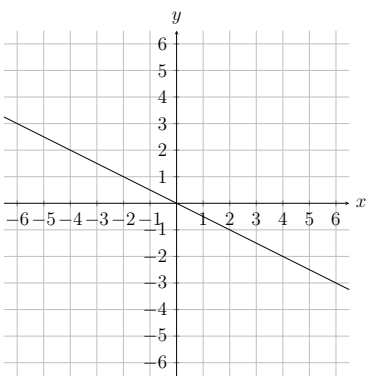
2. Sketch the graph of an *odd* function g such that

$$\lim_{x \to -2} g(x) = 1.$$

   There are *many* good solutions here, but we give two easy ones:





3. Evaluate

$$\lim_{x\to 2} \frac{\frac{2}{x} - \frac{x}{2}}{4 - x^2}.$$

We simplify the fraction and compute the remaining (simple) limit.

$$\begin{aligned}
\frac{\frac{2}{x} - \frac{x}{2}}{4 - x^2} &= \frac{\frac{2}{x} - \frac{x}{2}}{4 - x^2} \cdot \frac{2x}{2x} \\
&= \frac{4 - x^2}{(4 - x^2)(2x)} \\
&= \frac{1}{2x}.
\end{aligned}$$

Now, computing our limit is easy:

$$\begin{aligned}
\lim_{x\to 2} \frac{\frac{2}{x} - \frac{x}{2}}{4 - x^2} &= \lim_{x\to 2} \frac{1}{2x} \\
&= \frac{1}{2(2)} \\
&= \frac{1}{4}.
\end{aligned}$$

$\boxed{\frac{1}{4}}$