Name Solution

Quiz 4

Put a box around your answer and show all work!

1. Determine the value of K so that $f(x)$ is continuous at $x = 2$:

$$f(x) = \begin{cases} \sin(\pi x^2) & x \leq 2 \\ \pi x^2 - K & x > 2 \end{cases}$$

We compute the limit of f as x approaches 2 from the left and from the right and assign a value to K so that these limits agree.

$$\begin{aligned} \lim_{x \to 2^-} f(x) &= \lim_{x \to 2^-} \sin(\pi x^2) \\ &= \sin(\pi 2^2) \\ &= 0, \end{aligned}$$

and

$$\begin{aligned} \lim_{x \to 2^+} f(x) &= \lim_{x \to 2^+} \pi x^2 - K \\ &= \pi 2^2 - K \\ &= 4\pi - K. \end{aligned}$$

Now, for f to be continuous at $x = 2$, these limits should agree, meaning $0 = 4\pi - K$. Thus $K = 4\pi$.

$\boxed{K = 4\pi}$

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2. Use the Intermediate Value Theorem to show that there is a solution to $\sin(x) = x - 1$ in the interval $[0, \pi]$.

The Intermediate Value Theorem says that if a function f is continuous on $[a, b]$, then f attains every value between $f(a)$ and $f(b)$ on $[a, b]$. Thus, if $f(x) = \sin(x) - (x - 1) = \sin(x) - x + 1$, then f is continuous on $[0, \pi]$. Now, $f(0) = 1 > 0$ and $f(\pi) = 1 - \pi < 0$, so f must take on the value 0 at some c in $[0, \pi]$, meaning $f(c) = \sin(c) - (c-1) = 0$. Thus $\sin(c) = c - 1$, and we are done.

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3. Evaluate:

$$\lim_{x \to 0} \frac{\sin(x)}{2x}$$

Given that the limit of $\frac{\sin(x)}{x}$ is 1 as $x \to 0$, we have

$$\begin{aligned} \lim_{x \to 0} \frac{\sin(x)}{2x} &= \lim_{x \to 0} \frac{\sin(x)}{x} \cdot \frac{1}{2} \\ &= 1 \cdot \frac{1}{2} \\ &= \frac{1}{2}. \end{aligned}$$

$\boxed{\frac{1}{2}}$

Name Solution

Quiz 4

Put a box around your answer and show all work!

1. Determine the value of K so that $f(x)$ is continuous at $x = 0$:

$$f(x) = \begin{cases} \log(x + K) & x \leq 0 \\ 2x + 2 & x > 0 \end{cases}$$

We compute the limit of f as x approaches 0 from the left and from the right and assign a value to K so that these limits agree.

$$\begin{aligned} \lim_{x \to 0^-} f(x) &= \lim_{x \to 0^-} \log(x + K) \\ &= \log(K), \end{aligned}$$

and

$$\begin{aligned} \lim_{x \to 0^+} f(x) &= \lim_{x \to 0^+} 2x + 2 \\ &= 2. \end{aligned}$$

Now, for f to be continuous at $x = 0$, these limits should agree, meaning $2 = \log(K)$, and thus $K = 10^2 = 100$.

$\boxed{K = 100}$

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2. Use the Intermediate Value Theorem to show that there is a solution to $\cos(x) = 3x$ in the interval $[0, \pi]$.

The Intermediate Value Theorem says that if a function f is continuous on $[a, b]$, then f attains every value between $f(a)$ and $f(b)$ on $[a, b]$. Thus, if $f(x) = \cos(x) - 3x$, then f is continuous on $[0, \pi]$. Now, $f(0) = \cos(0) = 1 > 0$, and $f(\pi) = \cos(\pi) - 3\pi = -1 - 3\pi < 0$, so f must take on the value 0 at some c in $[0, \pi]$, meaning $f(c) = \cos(c) - 3c = 0$. Thus $\cos(c) = 3c$, and we are done.

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3. Evaluate:

$$\lim_{x \to 0} \frac{\sin(2x)}{x}$$

We know the limit of $\frac{\sin(x)}{x}$ is 1 as $x \to 0$, and as a consequence we know the limit of $\frac{\sin(2x)}{2x}$ is 1 as $x \to 0$ as well. Thus

$$\begin{aligned}\lim_{x \to 0} \frac{\sin(2x)}{x} &= \lim_{x \to 0} \frac{\sin(2x)}{2x} \cdot \frac{2}{1} \\ &= 1 \cdot \frac{2}{1} \\ &= 2.\end{aligned}$$

$\boxed{2}$