Name _____

Quiz 6

Put a box around your answer and show all work!

1. Find the equation of the tangent line of $f(x) = \sin(x)\cos(x)$ at $x = \pi$.

   First, we find the derivative of f at $x = \pi$:

$$\begin{aligned} f'(x) &= \cos(x) \cdot \frac{d}{dx}(\sin(x)) + \frac{d}{dx}(\cos(x)) \cdot \sin(x) \\ &= \cos(x)(\cos(x)) + (-\sin(x))(\sin(x)) \\ &= \cos^2(x) - \sin^2(x) \implies \\ f'(\pi) &= \cos^2(\pi) - \sin^2(\pi) \\ &= (-1)^2 - 0 \\ &= 1. \end{aligned}$$

   Now, we find the equation of our tangent line using the slope $f'(\pi) = 1$ and the point $(\pi, f(\pi)) = (\pi, 0)$:

$$\boxed{y - 0 = 1(x - \pi)}.$$

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2. Find the second derivative of $g(x) = x^3 + 3x^2 + 3x + 1$.

We take the derivative of g twice.

$$\begin{aligned} g'(x) &= \frac{d}{dx}\, g(x) \\ &= \frac{d}{dx}(x^3 + 3x^2 + 3x + 1) \\ &= 3x^2 + 6x + 3 \implies \\ g''(x) &= \frac{d}{dx}\, g'(x) \\ &= \frac{d}{dx}(3x^2 + 6x + 3) \\ &= \boxed{6x + 6}. \end{aligned}$$

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3. Find the derivative of $f(x) = \cot(x)\cos(x)$.

   We use the product rule:

$$\begin{aligned} f'(x) &= \cot(x)\,\frac{d}{dx}(\cos(x)) + \frac{d}{dx}(\cot(x))\cos(x) \\ &= \cot(x)(-\sin(x)) + (-\csc^2(x))\cos(x) \\ &= \boxed{-\cos(x) - \cot(x)\csc(x)}. \end{aligned}$$

Name _____

Quiz 6

Put a box around your answer and show all work!

1. Find the equation of the tangent line of $f(x) = e^x \sin(x)$ at $x = \frac{\pi}{2}$.

   First, we find the derivative of f at $x = \frac{\pi}{2}$:

$$\begin{aligned} f'(x) &= e^x \cdot \frac{d}{dx}(\sin(x)) + \frac{d}{dx}(e^x) \cdot \sin(x) \\ &= e^x(\cos(x)) + e^x(\sin(x)) \\ &= e^x(\cos(x) + \sin(x)) \implies \\ f'(\pi/2) &= e^{\pi/2}(\cos(\pi/2) + \sin(\pi/2)) \\ &= e^{\pi/2}(0 + 1) \\ &= e^{\pi/2}. \end{aligned}$$

   Now, we find the equation of our tangent line using the slope $f'(\pi/2) = e^{\pi/2}$ and the point $(\pi/2, e^{\pi/2})$.

$$\boxed{y - e^{\pi/2} = e^{\pi/2}(x - \pi/2)}.$$

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2. Find the second derivative of $g(x) = x^3 - 3x^2 + 3x - 1$.

   We take the derivative of g twice.

$$\begin{aligned} g'(x) &= \frac{d}{dx}\, g(x) \\ &= \frac{d}{dx}(x^3 - 3x^2 + 3x - 1) \\ &= 3x^2 - 6x + 3 \implies \\ g''(x) &= \frac{d}{dx}\, g'(x) \\ &= \frac{d}{dx}(3x^2 - 6x + 3) \\ &= \boxed{6x - 6}. \end{aligned}$$

3. Find the derivative of $f(x) = \tan(x)\sin(x)$.

We use the product rule:

$$\begin{aligned} f'(x) &= \tan(x)\frac{d}{dx}(\sin(x)) + \frac{d}{dx}(\tan(x))\sin(x) \\ &= \tan(x)\cos(x) + \sec^2(x)\sin(x) \\ &= \boxed{\sin(x) + \sec(x)\tan(x)}. \end{aligned}$$