Name Solution

Quiz 7

Put a box around your answer and show all work!

1. Find each x-value at which the graph of $f(x) = x\sqrt[3]{2x+3}$ has a horizontal tangent line.

   A horizontal tangent line means a derivative equal to 0, so we first take the derivative:

$$\begin{aligned} f'(x) &= x\,\frac{d}{dx}\left(\sqrt[3]{2x+3}\right) + \sqrt[3]{2x+3}\,\frac{d}{dx}(x) \\ &= x\left(\frac{1}{3}(2x+3)^{-2/3}(2)\right) + (2x+3)^{1/3} \\ &= \frac{\frac{2}{3}x}{(2x+3)^{2/3}} + \frac{(2x+3)^1}{(2x+3)^{2/3}} \\ &= \frac{\frac{8}{3}x+3}{(2x+3)^{2/3}}. \end{aligned}$$

   Now, we simply set our derivative equal to 0 and solve for x.

$$\begin{aligned} \frac{\frac{8}{3}x+3}{(2x+3)^{2/3}} &= 0 \implies \\ \frac{8}{3}x+3 &= 0 \implies \\ \frac{8}{3}x &= -3 \implies \\ x &= -\frac{9}{8}. \end{aligned}$$

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2. If $g(x)$ is the inverse function of $f(x) = e^{3x} + x$, find $g'(1)$.

   According to the 'inverse function theorem', we have that

$$g'(1) = \frac{1}{f'(g(1))}.$$

Now, we try some 'easy' values to see that $f(0) = 1$, so $g(1) = f^{-1}(1) = 0$. We also find the derivative of f,

$$f'(x) = 3e^{3x} + 1,$$

so $f'(g(1)) = f'(0) = 3e^0 + 1 = 4$, and

$$g'(1) = \frac{1}{f'(g(1))} = \frac{1}{4}.$$

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3. Find $\frac{dy}{dx}$ if $y = x^{\sin(x)}$ for $x > 0$.

   We use logarithmic differentiation.

$$\begin{aligned}
y &= x^{\sin(x)} \implies \\
\ln(y) &= \ln(x^{\sin(x)}) \implies \\
\ln(y) &= \sin(x) \cdot \ln(x) \implies \\
\frac{\left(\frac{dy}{dx}\right)}{y} &= \sin(x) \cdot \frac{1}{x} + \ln(x) \cdot \cos(x) \implies \\
\frac{dy}{dx} &= \left(\frac{\sin(x)}{x} + \ln(x)\cos(x)\right) y.
\end{aligned}$$

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1. Find each x-value at which the graph of $f(x) = 3x\sqrt[3]{x+1}$ has a horizontal tangent line.

   A horizontal tangent line means a derivative equal to 0, so we must first take the derivative:

$$\begin{aligned} f'(x) &= 3x\,\frac{d}{dx}\left(\sqrt[3]{x+1}\right) + \sqrt[3]{x+1}\,\frac{d}{dx}(3x) \\ &= 3x\left(\frac{1}{3}(x+1)^{-2/3}(1)\right) + \sqrt[3]{x+1}(3) \\ &= \frac{x}{(x+1)^{2/3}} + \frac{(x+1)^1}{(x+1)^{2/3}} \\ &= \frac{2x+1}{(x+1)^{2/3}} \end{aligned}$$

   Now, we simplify set our derivative equal to 0 and solve for x.

$$\begin{aligned} \frac{2x+1}{(x+1)^{2/3}} = 0 &\implies \\ 2x+1 = 0 &\implies \\ x = -\frac{1}{2} \end{aligned}$$

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2. If $g(x)$ is the inverse function of $f(x) = e^x + 3x$, find $g'(1)$.

   According to the 'inverse function theorem', we have that

$$g'(1) = \frac{1}{f'(g(1))}.$$

   Now, we try some 'easy' values to see that $f(0) = 1$, so $g(1) = f^{-1}(1) = 0$. We also find the derivative of f,

$$f'(x) = e^x + 3,$$

so $f'(g(1)) = f'(0) = e^0 + 3 = 4$¡ and

$$g'(1) = \frac{1}{f'(g(1))} = \frac{1}{4}.$$

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3. Find $\dfrac{dy}{dx}$ if $y = (\cos(x))^{-x}$ for $x > 0$.

   We use logarithmic differentiation.

$$\begin{aligned}
y &= (\cos(x))^{-x} \implies \\
\ln(y) &= \ln((\cos(x))^{-x}) \implies \\
\ln(y) &= (-x)\ln(\cos(x)) \implies \\
\frac{\left(\frac{dy}{dx}\right)}{y} &= (-x)\frac{-\sin(x)}{\cos(x)} + (-1)\ln(\cos(x)) \implies \\
\frac{dy}{dx} &= \left(x\frac{\sin(x)}{\cos(x)} - \ln(\cos(x))\right)y.
\end{aligned}$$