Name Solution

Quiz 8

Put a box around your answer and show all work!

1. Find the linearization of $f(x) = \sqrt[3]{x^2}$ at $x = 8$ and use it to approximate $\sqrt[3]{8.06^2}$.

   The formula for the linearization $L(x)$ of f near $x = 8$ is given by

   $$L(x) = f'(8) \cdot (x - 8) + f(8).$$

   In this case,

   $$f'(x) = \frac{2}{3x^{1/3}},$$

   so $f'(8) = \frac{2}{3 \cdot 2} = 1/3$ and $f(8) = 4$, so our linearization is

   $$L(x) = \frac{1}{3}(x - 8) + 4.$$

   To estimate $f(8.06) = \sqrt[3]{8.06^2}$, we simply plug 8.06 into our linearization:

   $$L(8.06) = \frac{1}{3}(8.06 - 8) + 4 = \frac{1}{3}(0.06) + 4 = 4.02.$$

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2. Find the absolute extrema of $g(x) = \sin(x) + \cos(x)$ on $[0, \pi]$.

   We find the critical points of g, then compare the function values at these critical points and the endpoints of the interval. The critical points are the points x in the domain of g where $g'(x) = 0$ or $g'(x)$ is undefined. $g'(x) = \cos(x) - \sin(x)$, so

   $$g'(x) = \cos(x) - \sin(x) = 0 \implies$$
   $$\cos(x) = \sin(x) \implies$$
   $$x = \frac{\pi}{4}.$$

   Now, we consider the values of g at the critical value and the endpoints of the interval:

   $$g(0) = 1, \quad g(\pi/4) = \sqrt{2}, \quad g(\pi) = -1.$$

The largest of these values, $g(\pi/4) = \sqrt{2}$ is our absolute maximum, and the smallest of these values, $g(\pi) = -1$ is our absolute minimum.

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3. Find the value of c implied by the Mean Value Theorem for $h(x) = x^3 - 3x$ on $[1, 3]$.

   First, we find the slope of the secant line connecting $(1, f(1))$ and $(3, f(3))$. The slope is given by

$$\begin{aligned} \frac{f(3) - f(1)}{3 - 1} &= \frac{(3^3 - 3 \cdot 3) - (1^3 - 3 \cdot 1)}{2} \\ &= \frac{20}{2} \\ &= 10. \end{aligned}$$

   Now, we find the value c in $(1, 3)$ so that $f'(c) = 10$. The derivative of f is $f'(x) = 3x^2 - 3$, so

$$\begin{aligned} 3c^2 - 3 = 10 &\implies \\ 3c^2 = 13 &\implies \\ c^2 = \frac{13}{3} &\implies \\ c = \pm\sqrt{\frac{13}{3}}. \end{aligned}$$

   Now, we want the positive solution as c should lie in $[1, 3]$.

   $\boxed{c = \sqrt{13/3}}$

Name Solution

Quiz 8

Put a box around your answer and show all work!

1. Find the linearization of $f(x) = \sqrt[3]{x}$ at $x = -8$ and use it to approximate $\sqrt[3]{-7.88}$.

   The formula for the linearization $L(x)$ of f near $x = -8$ is given by
$$L(x) = f'(-8) \cdot (x - (-8)) + f(-8).$$
   In this case,
$$f'(x) = \frac{1}{3x^{2/3}},$$
   so $f'(-8) = \frac{1}{3 \cdot 4} = 1/12$ and $f(-8) = -2$, so our linearization is
$$L(x) = \frac{1}{12}(x+8) - 2.$$
   To estimate $f(-7.88) = \sqrt[3]{-7.88}$, we simply plug -7.88 into our linearization:
$$L(-7.88) = \frac{1}{12}(-7.88+8) - 2 = \frac{0.12}{12} - 2 = -1.99.$$

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2. Find the absolute extrema of $g(x) = \sin(x) - \cos(x)$ on $[0, \pi]$.

   We find the critical points of g, then compare the function values at these critical points and the endpoints of the interval. The critical points are the points x in the domain of g where $g'(x) = 0$ or $g'(x)$ is undefined. $g'(x) = \cos(x) + \sin(x)$, so
$$g'(x) = \cos(x) + \sin(x) = 0 \implies$$
$$\cos(x) = -\sin(x) \implies$$
$$x = \frac{3\pi}{4}.$$

Now, we consider the values of g at the critical value and the endpoints of the interval:

$$g(0) = 1, \quad g(3\pi/4) = \sqrt{2}, \quad g(\pi) = -1.$$

The largest of these values, $g(3\pi/4) = \sqrt{2}$ is our absolute maximum, and the smallest of these values, $g(\pi) = -1$ is our absolute minimum.

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3. Find the value of c implied by the Mean Value Theorem for $h(x) = x^2 - 2x + 4$ on $[0, 5]$.

   First, we find the slope of the secant line connecting $(0, f(0))$ and $(5, f(5))$. The slope is given by

$$\begin{aligned} \frac{f(5) - f(0)}{5 - 0} &= \frac{(5^2 - 2 \cdot 5 + 4) - (0^2 - 2 \cdot 0 + 4)}{5 - 0} \\ &= \frac{15}{5} \\ &= 3. \end{aligned}$$

   Now, we find the value c in $(1, 3)$ so that $f'(c) = 3$. The derivative of f is $f'(x) = 2x - 2$, so

$$\begin{aligned} 2c - 2 = 3 &\implies \\ 2c = 5 &\implies \\ c = 5/2&. \end{aligned}$$

   $\boxed{c = 5/2}$