1. Let $f(x) = \dfrac{1}{x-2}$ and $g(x) = \dfrac{x}{x+1}$. Find $(g \circ f)(x)$ and its domain.

   Solution. First, we find the composition $(g \circ f)(x) = g(f(x))$ by plugging the formula for $f(x)$ into the formula for $g(x)$:

$$\begin{aligned} g(f(x)) &= g\left(\frac{1}{x-2}\right) \\ &= \frac{\left(\frac{1}{x-2}\right)}{\left(\frac{1}{x-2}\right)+1} \\ &= \frac{\left(\frac{1}{x-2}\right)}{\left(\frac{1}{x-2}\right)+1} \cdot \frac{x-2}{x-2} \\ &= \frac{1}{1+(x-2)} \\ &= \frac{1}{x-1}. \end{aligned}$$

   Now, to find the domain of $(g \circ f)(x)$, we consider both the domain of $\dfrac{1}{x-1}$ and the domain of $f(x)$. The domain of $\dfrac{1}{x-1}$ includes every value except $x = 1$, and written in interval notation is $(-\infty, 1) \cup (1, \infty)$. The domain of $f(x) = \dfrac{1}{x-2}$ includes every value except $x = 2$, and written in interval notation is $(-\infty, 1) \cup (1, \infty)$.

   The domain of the composition $(g \circ f)(x)$ is the *intersection* of these domains, and thus is

$$\boxed{(-\infty, 1) \cup (1, 2) \cup (2, \infty).}$$

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2. Solve for x:

$$\frac{3}{x+1} + \frac{1}{x-1} = 2$$

   Solution. First, we combine the two terms on the lefthand side using a common denominator:

$$\begin{aligned}
\frac{3}{x+1}+\frac{1}{x-1} &= 2 \\
\frac{3}{x+1}\cdot\frac{x-1}{x-1}+\frac{1}{x-1}\cdot\frac{x+1}{x+1} &= 2 \\
\frac{3x-3}{(x+1)(x-1)}+\frac{x+1}{(x+1)(x-1)} &= 2 \\
\frac{4x-2}{x^2-1} &= 2.
\end{aligned}$$

Now, we multiply both sides by x^2-1 to eliminate the denominator.

$$\begin{aligned}
(x^2-1)\cdot\frac{4x-2}{x^2-1} &= 2\cdot(x^2-1) \\
4x-2 &= 2x^2-2.
\end{aligned}$$

Gathering the terms on one side and factoring lets us see the solutions.

$$\begin{aligned}
4x-2 &= 2x^2-2 \\
0 &= 2x^2-4x \\
0 &= 2x(x-2).
\end{aligned}$$

The expression $2x(x-2)$ is zero when either $x=0$ or $x-2=0$, meaning our solutions are at $x=0$ and $x=2$. The domain of our original equation only disallows $x=-1$ and $x=1$, so both $x=0$ and $x=2$ are 'good' solutions.

$$\boxed{x=0,2.}$$

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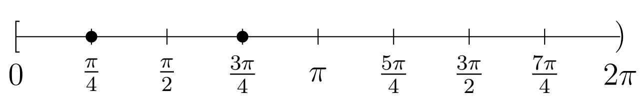
3. Find all values of θ in $[0,2\pi)$ such that $2\sin\theta \geq \sqrt{2}$. Write your answer in interval notation.

Solution. First, we find the values of θ so that $2\sin(\theta)=\sqrt{2}$.

$$\begin{aligned}
2\sin(\theta) &= \sqrt{2} \\
\sin(\theta) &= \frac{\sqrt{2}}{2}
\end{aligned}$$

The sine function takes the value $\frac{\sqrt{2}}{2}$ when $\theta=\pi/4$ and $\theta=3\pi/4$.



Now, we simply check points in each 'region' of the number line, $[0, \pi/4)$, $(\pi/4, 3\pi/4)$, and $(3\pi/4, 2\pi)$. The regions we want to include are those where the sample point yields a 'good' result, i.e. $\sin(\theta) \geq \frac{\sqrt{2}}{2}$.

$$
\begin{aligned}
\sin(0) &= 0 \\
&\ngeq \frac{\sqrt{2}}{2}, \text{ so } [0, \pi/4) \text{ is not included.} \\
\sin(\pi/2) &= 1 \\
&\geq \frac{\sqrt{2}}{2}, \text{ so } (\pi/4, 3\pi/4) \text{ is included.} \\
\sin(\pi) &= 0 \\
&\ngeq \frac{\sqrt{2}}{2}, \text{ so } (3\pi/4, 2\pi) \text{ is not included.}
\end{aligned}
$$

Lastly, we also choose to include $\theta = \pi/4, 3\pi/4$ because they do satisfy the original equation. Our answer is

$$\boxed{[\pi/4, 3\pi/4].}$$