1. Let $f(x) = \sqrt{4 - x^2}$ and $g(x) = \dfrac{1}{x^2 - 4}$. Find $(g \circ f)(x)$ and its domain.

   Solution. First, we find the composition $(g \circ f)(x) = g(f(x))$ by plugging the formula for $f(x)$ into the formula for $g(x)$:

$$\begin{aligned} g(f(x)) &= g\left(\sqrt{4 - x^2}\right) \\ &= \frac{1}{(\sqrt{4 - x^2})^2 - 4} \\ &= \frac{1}{(4 - x^2) - 4} \\ &= \frac{1}{-x^2}. \end{aligned}$$

   Now, to find the domain of $(g \circ f)(x)$, we consider both the domain of $\dfrac{1}{-x^2}$ and the domain of $f(x)$. The domain of $\dfrac{1}{-x^2}$ includes every value except $x = 0$, and written in interval notation is $(-\infty, 0) \cup (0, \infty)$. The domain of $f(x) = \sqrt{4 - x^2}$ is determined by finding where the *radicand* $4 - x^2$ is at least 0, that is

$$\begin{aligned} 4 - x^2 &\geq 0 \\ 4 &\geq x^2. \end{aligned}$$

   If $x^2 \leq 4$, then $|x| \leq 2$, and so the domain of f is $[-2, 2]$.

   The domain of the composition $(g \circ f)(x)$ is the *intersection* of these domains, and thus is

$$\boxed{[-2, 0) \cup (0, 2].}$$

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2. Solve for x:

$$\frac{1}{x + 1} + \frac{3}{x - 1} + 2 = 0$$

   Solution. First, we move the constant term to the righthand side and combine the two remaining terms on the lefthand side using a common denominator:

$$\begin{aligned}
\frac{1}{x+1}+\frac{3}{x-1}+2 &= 0\\
\frac{1}{x+1}+\frac{3}{x-1} &= -2\\
\frac{1}{x+1}\cdot\frac{x-1}{x-1}+\frac{3}{x-1}\cdot\frac{x+1}{x+1} &= -2\\
\frac{x-1}{(x+1)(x-1)}+\frac{3x+3}{(x+1)(x-1)} &= -2\\
\frac{4x+2}{x^2-1} &= -2.
\end{aligned}$$

Now, we multiply both sides by x^2-1 to eliminate the denominator.

$$\begin{aligned}
(x^2-1)\cdot\frac{4x+2}{x^2-1} &= -2\cdot(x^2-1)\\
4x+2 &= -2x^2+2.
\end{aligned}$$

Gathering the terms on one side and factoring lets us see the solutions.

$$\begin{aligned}
4x+2 &= -2x^2+2\\
2x^2+4x &= 0\\
2x(x+2) &= 0.
\end{aligned}$$

The expression $2x(x+2)$ is zero when either $x=0$ or $x+2=0$, meaning our solutions are at $x=0$ and $x=-2$. The domain of our original equation only disallows $x=-1$ and $x=1$, so both $x=0$ and $x=-2$ are 'good' solutions.

$$\boxed{x=-2,0.}$$

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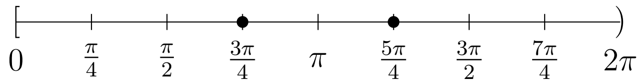
3. Find all values of θ in $[0,2\pi)$ such that $2\cos\theta+\sqrt{2}\le 0$. Write your answer in interval notation.

Solution. First, we find the values of θ so that $2\cos(\theta)+\sqrt{2}=0$.

$$\begin{aligned}
2\cos(\theta)+\sqrt{2} &= 0\\
2\cos(\theta) &= -\sqrt{2}\\
\cos(\theta) &= -\frac{\sqrt{2}}{2}.
\end{aligned}$$

The cosine function takes the value $-\frac{\sqrt{2}}{2}$ when $\theta=3\pi/4$ and $\theta=5\pi/4$.



Now, we simply check points in each 'region' of the number line, $[0, 3\pi/4)$, $(3\pi/4, 5\pi/4)$, and $(5\pi/4, 2\pi)$. The regions we want to include are those where the sample point yields a 'good' result, i.e. $\cos(\theta) \leq -\frac{\sqrt{2}}{2}$.

$$\begin{aligned} \cos(0) &= 1 \\ &\not\leq -\frac{\sqrt{2}}{2}, \text{ so } [0, 3\pi/4) \text{ is not included.} \\ \cos(\pi) &= -1 \\ &\leq \frac{\sqrt{2}}{2}, \text{ so } (3\pi/4, 5\pi/4) \text{ is included.} \\ \cos(3\pi/2) &= 0 \\ &\not\leq \frac{\sqrt{2}}{2}, \text{ so } (3\pi/4, 2\pi) \text{ is not included.} \end{aligned}$$

Lastly, we also choose to include $\theta = 3\pi/4, 5\pi/4$ because they do satisfy the original equation. Our answer is

$$\boxed{[3\pi/4, 5\pi/4].}$$