1. Let \( f(x) = \sqrt{4 - x^2} \) and \( g(x) = \frac{1}{x^2 - 4} \). Find \((g \circ f)(x)\) and its domain.

**Solution.** First, we find the composition \((g \circ f)(x) = g(f(x))\) by plugging the formula for \(f(x)\) into the formula for \(g(x)\):

\[
g(f(x)) = g\left(\sqrt{4 - x^2}\right)
= \frac{1}{(\sqrt{4 - x^2})^2 - 4}
= \frac{1}{4 - x^2 - 4}
= \frac{1}{-x^2}.
\]

Now, to find the domain of \((g \circ f)(x)\), we consider both the domain of \(\frac{1}{-x^2}\) and the domain of \(f(x)\). The domain of \(\frac{1}{-x^2}\) includes every value except \(x = 0\), and written in interval notation is \((-\infty, 0) \cup (0, \infty)\). The domain of \(f(x) = \sqrt{4 - x^2}\) is determined by finding where the radicand \(4 - x^2\) is at least 0, that is

\[
4 - x^2 \geq 0
\]

\[
x^2 \leq 4
\]

If \(x^2 \leq 4\), then \(|x| \leq 2\), and so the domain of \(f\) is \([-2, 2]\). The domain of the composition \((g \circ f)(x)\) is the *intersection* of these domains, and thus is

\[
[-2, 0) \cup (0, 2].
\]

2. Solve for \(x\):

\[
\frac{1}{x + 1} + \frac{3}{x - 1} + 2 = 0
\]

**Solution.** First, we move the constant term to the righthand side and combine the
two remaining terms on the lefthand side using a common denominator:

\[
\frac{1}{x+1} + \frac{3}{x-1} + 2 = 0
\]

\[
\frac{1}{x+1} + \frac{3}{x-1} = -2
\]

\[
\frac{1}{x+1} \cdot \frac{x-1}{x-1} + \frac{3}{x-1} \cdot \frac{x+1}{x+1} = -2
\]

\[
\frac{x-1}{(x+1)(x-1)} + \frac{3x+3}{(x+1)(x-1)} = -2
\]

\[
\frac{4x+2}{x^2-1} = -2.
\]

Now, we multiply both sides by \(x^2 - 1\) to eliminate the denominator.

\[
(x^2 - 1) \cdot \frac{4x+2}{x^2-1} = -2 \cdot (x^2 - 1)
\]

\[
4x + 2 = -2x^2 + 2.
\]

Gathering the terms on one side and factoring lets us see the solutions.

\[
4x + 2 = -2x^2 + 2
\]

\[
2x^2 + 4x = 0
\]

\[
2x(x + 2) = 0.
\]

The expression \(2x(x + 2)\) is zero when either \(x = 0\) or \(x + 2 = 0\), meaning our solutions are at \(x = 0\) and \(x = -2\). The domain of our original equation only disallows \(x = -1\) and \(x = 1\), so both \(x = 0\) and \(x = -2\) are ‘good’ solutions.

\[
x = -2, 0.
\]

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3. Find all values of \(\theta\) in \([0, 2\pi)\) such that \(2 \cos \theta + \sqrt{2} \leq 0\). Write your answer in interval notation.

**Solution.** First, we find the values of \(\theta\) so that \(2 \cos(\theta) + \sqrt{2} = 0\).

\[
2 \cos(\theta) + \sqrt{2} = 0
\]

\[
2 \cos(\theta) = -\sqrt{2}
\]

\[
\cos(\theta) = -\frac{\sqrt{2}}{2}.
\]

The cosine function takes the value \(-\frac{\sqrt{2}}{2}\) when \(\theta = 3\pi/4\) and \(\theta = 5\pi/4\).
Now, we simply check points in each ‘region’ of the number line, $[0, 3\pi/4)$, $(3\pi/4, 5\pi/4)$, and $(5\pi/4, 2\pi)$. The regions we want to include are those where the sample point yields a ‘good’ result, i.e. $\cos(\theta) \leq -\frac{\sqrt{2}}{2}$.

\[
\cos(0) = 1 \\
\n\not< -\frac{\sqrt{2}}{2}, \text{ so } [0, 3\pi/4) \text{ is not included.}
\]

\[
\cos(\pi) = -1 \\
\n\leq \frac{\sqrt{2}}{2}, \text{ so } (3\pi/4, 5\pi/4) \text{ is included.}
\]

\[
\cos(3\pi/2) = 0 \\
\n\not< \frac{\sqrt{2}}{2}, \text{ so } (3\pi/4, 2\pi) \text{ is not included.}
\]

Lastly, we also choose to include $\theta = 3\pi/4, 5\pi/4$ because they do satisfy the original equation. Our answer is

\[
[3\pi/4, 5\pi/4].
\]