1. Determine the value of $K$ so that $f(x)$ is continuous at $x = 2$:

$$f(x) = \begin{cases} \sin(\pi x^2) & x \le 2 \\ \pi x^2 - K & x > 2 \end{cases}$$

**Solution.** We compute the limit of $f$ as $x$ approaches 2 from the left and from the right and assign a value to $K$ so that these limits agree.

$$\begin{aligned} \lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} \sin(\pi x^2) \\ &= \sin(\pi 2^2) \\ &= 0, \end{aligned}$$

and

$$\begin{aligned} \lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} \pi x^2 - K \\ &= \pi 2^2 - K \\ &= 4\pi - K. \end{aligned}$$

Now, for $f$ to be continuous at $x = 2$, these limits should agree, meaning $0 = 4\pi - K$. Thus $K = 4\pi$.

$$\boxed{K = 4\pi}$$

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2. Use the Intermediate Value Theorem to show that there is a solution to $\sin(x) = x - 1$ in the interval $[0, \pi]$.

**Solution.** The Intermediate Value Theorem says that if a function $f$ is continuous on $[a, b]$, then $f$ attains every value between $f(a)$ and $f(b)$ on $[a, b]$. Thus, if

$$f(x) = \sin(x) - (x - 1) = \sin(x) - x + 1,$$

then $f$ is continuous on $[0, \pi]$. Now, $f(0) = 1 > 0$ and $f(\pi) = 1 - \pi < 0$, so $f$ must take on the value 0 at some $c$ in $[0, \pi]$, meaning $f(c) = \sin(c) - (c - 1) = 0$. Thus $\sin(c) = c - 1$, and we are done.

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3. Evaluate

$$\lim_{x\to 2} \frac{\frac{1}{x} - \frac{1}{2}}{2 - x}.$$

**Solution.** We simplify the fraction and compute the remaining (simple) limit.

$$\begin{aligned}\frac{\frac{2}{x}-\frac{x}{2}}{4-x^2} &= \frac{\frac{2}{x}-\frac{x}{2}}{4-x^2}\cdot\frac{2x}{2x}\\ &= \frac{4-x^2}{(4-x^2)(2x)}\\ &= \frac{1}{2x}.\end{aligned}$$

Now, computing our limit is easy:

$$\begin{aligned}\lim_{x\to 2}\frac{\frac{2}{x}-\frac{x}{2}}{4-x^2} &= \lim_{x\to 2}\frac{1}{2x}\\ &= \frac{1}{2(2)}\\ &= \frac{1}{4}.\end{aligned}$$

$$\boxed{\frac{1}{4}}$$