1. Determine the value of K so that $f(x)$ is continuous at $x = 0$:

$$f(x) = \begin{cases} \log(x+K) & x \leq 0 \\ 2x+2 & x > 0 \end{cases}$$

Solution. We compute the limit of f as x approaches 0 from the left and from the right and assign a value to K so that these limits agree.

$$\begin{aligned} \lim_{x\to 0^-} f(x) &= \lim_{x\to 0^-} \log(x+K) \\ &= \log(K), \end{aligned}$$

and

$$\begin{aligned} \lim_{x\to 0^+} f(x) &= \lim_{x\to 0^+} 2x+2 \\ &= 2. \end{aligned}$$

Now, for f to be continuous at $x = 0$, these limits should agree, meaning $2 = \log(K)$, and thus $K = 10^2 = 100$.

$$\boxed{K = 100}$$

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2. Use the Intermediate Value Theorem to show that there is a solution to $\cos(x) = 3x$ in the interval $[0, \pi]$.

Solution. The Intermediate Value Theorem says that if a function f is continuous on $[a, b]$, then f attains every value between $f(a)$ and $f(b)$ on $[a, b]$. Thus, if

$$f(x) = \cos(x) - 3x,$$

then f is continuous on $[0, \pi]$. Now, $f(0) = \cos(0) = 1 > 0$, and $f(\pi) = \cos(\pi) - 3\pi = -1 - 3\pi < 0$, so f must take on the value 0 at some c in $[0, \pi]$, meaning $f(c) = \cos(c) - 3c = 0$. Thus $\cos(c) = 3c$, and we are done.

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3. Evaluate

$$\lim_{x\to 2} \frac{\frac{2}{x} - \frac{x}{2}}{4 - x^2}.$$

Solution. We simplify the given fraction and compute the remaining (simple) limit.

$$\begin{aligned} \frac{\frac{1}{x} - \frac{1}{2}}{2 - x} &= \frac{\frac{1}{x} - \frac{1}{2}}{2 - x} \cdot \frac{2x}{2x} \\ &= \frac{2 - x}{(2-x)(2x)} \\ &= \frac{1}{2x}. \end{aligned}$$

Now, computing our limit is easy:

$$\begin{aligned}\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{2-x} &= \lim_{x\to 2}\frac{1}{2x}\\ &= \frac{1}{2(2)}\\ &= \frac{1}{4}.\end{aligned}$$

$$\boxed{\frac{1}{4}}$$