

1. Evaluate:

$$\lim_{x \rightarrow 0} \frac{\sin(2x)}{x}$$

Solution. We know the limit of $\frac{\sin(x)}{x}$ is 1 as $x \rightarrow 0$, and as a consequence we know the limit of $\frac{\sin(2x)}{2x}$ is 1 as $x \rightarrow 0$ as well. Thus

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin(2x)}{x} &= \lim_{x \rightarrow 0} \frac{\sin(2x)}{2x} \cdot \frac{2}{1} \\ &= 1 \cdot \frac{2}{1} \\ &= 2. \end{aligned}$$

$\boxed{2}$

2. Find the second derivative of $g(x) = x^3 - 3x^2 + 3x - 1$.

Solution. We take the derivative of g twice.

$$\begin{aligned} g'(x) &= \frac{d}{dx} g(x) \\ &= \frac{d}{dx} (x^3 - 3x^2 + 3x - 1) \\ &= 3x^2 - 6x + 3 \implies \\ g''(x) &= \frac{d}{dx} g'(x) \\ &= \frac{d}{dx} (3x^2 - 6x + 3) \\ &= 6x - 6. \end{aligned}$$

$\boxed{6x - 6}$

3. Find the derivative of $f(x) = \tan(x) \sin(x)$.

Solution. We use the product rule:

$$\begin{aligned} f'(x) &= \tan(x) \frac{d}{dx}(\sin(x)) + \frac{d}{dx}(\tan(x)) \sin(x) \\ &= \tan(x) \cos(x) + \sec^2(x) \sin(x) \\ &= \boxed{\sin(x) + \sec(x) \tan(x)} \\ &= \boxed{\frac{\sin(x)(\cos^2(x) + 1)}{\cos^2(x)}} \\ &= \boxed{\sin(x)(1 + \sec^2(x))}. \end{aligned}$$