1. Find the slope of the tangent line to $f(x) = \arccos(x)\sqrt{1-x^2}$ at $x = 0$.

Solution. The slope of the tangent line to $f(x)$ at $x = 0$ is defined as $f'(0)$, so we begin by differentiating using the product and chain rules:

$$\begin{aligned} f'(x) &= \arccos(x) \cdot \frac{1}{2}(-2x)(1-x^2)^{-1/2} + \sqrt{1-x^2} \cdot \frac{-1}{\sqrt{1-x^2}} \\ &= -\frac{x\arccos(x)}{\sqrt{1-x^2}} - 1. \end{aligned}$$

Now, we simply plug in $x = 0$:

$$f'(0) = -\frac{0\arccos(0)}{\sqrt{1-0^2}} - 1 = -1.$$

$$\boxed{f'(0) = -1}$$

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2. Find $\frac{dy}{dx}$ in terms of x and y if $e^{y/x} = \cos(x) + \sin(y)$.

Solution. We use implicit differentiation. Taking a derivative of either side gives

$$e^{y/x} \cdot \frac{x\frac{dy}{dx} - y}{x^2} = -\sin(x) + \tfrac{dy}{dx}\cos(y),$$

and some rearranging yields

$$\begin{aligned} \tfrac{dy}{dx}\left(xe^{y/x} - x^2\cos(y)\right) &= ye^{y/x} - x^2\sin(x) \implies \\ \frac{dy}{dx} &= \frac{ye^{y/x} - x^2\sin(x)}{xe^{y/x} - x^2\cos(y)}, \end{aligned}$$

which is our final answer.

$$\boxed{\frac{dy}{dx} = \frac{ye^{y/x} - x^2\sin(x)}{xe^{y/x} - x^2\cos(y)}}$$

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3. Find the derivative of $f(x) = (\sec(x))^x$ in terms of x.

Solution. We use logarithmic differentiation. Let $y = (\sec(x))^x$. Then

$$\ln(y) = \ln((\sec(x))^x) = x\ln(\sec(x)),$$

and taking a derivative gives

$$\begin{aligned}
\frac{\left(\frac{dy}{dx}\right)}{y} &= \ln(\sec(x)) + x\frac{\sec(x)\tan(x)}{\sec(x)} \\
&= \ln(\sec(x)) + x\tan(x) \implies \\
\frac{dy}{dx} &= y(\ln(\sec(x)) + x\tan(x)) \\
&= (\sec(x))^x(\ln(\sec(x)) + x\tan(x)).
\end{aligned}$$

$$\boxed{\frac{dy}{dx} = (\sec(x))^x(\ln(\sec(x)) + x\tan(x))}$$