1. Find the slope of the tangent line to $f(x) = x\sec^{-1}(x^2)$ at $x = \sqrt{2}$.

   Solution. The slope of the tangent line to $f(x)$ at $x = \sqrt{2}$ is defined as $f'(\sqrt{2})$, so we begin by differentiating using the product and chain rules:

$$\begin{aligned} f'(x) &= \sec^{-1}(x^2)\cdot 1 + x\cdot\frac{2x}{x^2\sqrt{x^4-1}} \\ &= \sec^{-1}(x^2) + \frac{2}{\sqrt{x^4-1}}. \end{aligned}$$

   Now, we simply plug in $x = \sqrt{2}$:

$$f'(\sqrt{2}) = \sec^{-1}(2) + \frac{2}{\sqrt{4-1}} = \frac{\pi}{3} + \frac{2}{\sqrt{3}}$$

$$\boxed{f'(\sqrt{2}) = \frac{\pi}{3} + \frac{2}{\sqrt{3}}}$$

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2. Find $\frac{dy}{dx}$ in terms of x and y if $e^{x/y} = \cos(y) + \sin(x)$.

   Solution. We use implicit differentiation. Taking a derivative of either side gives

$$e^{x/y}\cdot\frac{y - x\frac{dy}{dx}}{y^2} = -\tfrac{dy}{dx}\sin(y) + \cos(x),$$

   and some rearranging yields

$$\begin{aligned} \tfrac{dy}{dx}\left(y^2\sin(y) - xe^{x/y}\right) &= y^2\cos(x) - ye^{x/y} \implies \\ \frac{dy}{dx} &= \frac{y^2\cos(x) - ye^{x/y}}{y^2\sin(y) - xe^{x/y}}, \end{aligned}$$

   which is our final answer.

$$\boxed{\frac{dy}{dx} = \frac{y^2\cos(x) - ye^{x/y}}{y^2\sin(y) - xe^{x/y}}}$$

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3. Find the derivative of $f(x) = x^{\sin(x)}$ in terms of x.

   Solution. We use logarithmic differentiation. Let $y = x^{\sin(x)}$. Then

$$\ln(y) = \ln(x^{\sin(x)}) = \sin(x)\ln(x),$$

and taking a derivative gives

$$\begin{aligned}
\frac{\left(\frac{dy}{dx}\right)}{y} &= \cos(x)\ln(x) + \sin(x)\frac{1}{x} \implies \\
\frac{dy}{dx} &= y\left(\cos(x)\ln(x) + \frac{\sin(x)}{x}\right) \\
&= x^{\sin(x)}\left(\cos(x)\ln(x) + \frac{\sin(x)}{x}\right).
\end{aligned}$$

$$\boxed{\frac{dy}{dx} = x^{\sin(x)}\left(\cos(x)\ln(x) + \tfrac{\sin(x)}{x}\right)}$$