Name _____

Quiz 5

1. Using the definition, find the equation of the tangent line of $f(x) = 2x^3 + 2$ at $x = 3$. The slope of the tangent line is

$$m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

where here $x = 3$. So

$$\begin{aligned} m_{tan} &= \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} \\ &= \lim_{h \to 0} \frac{2(3+h)^3 + 2 - 2(3)^3 - 2}{h} \\ &= \lim_{h \to 0} \frac{2h^3 + 18h^2 + 54h}{h} \\ &= \lim_{h \to 0} (2h^2 + 18h + 54) \\ &= 54 \end{aligned}$$

The tangent line also goes through the point $(3, f(3) = (3, 56)$, so the equation (using point-slope form) is

$$y - 56 = 54(x - 3).$$

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2. Find the derivatives of the given functions:

(a) $$\begin{aligned} f'(x) &= \frac{(x-2)\frac{\mathrm{d}}{\mathrm{dx}}(x^3 - 8) - (x^3 - 8)\frac{\mathrm{d}}{\mathrm{dx}}(x-2)}{(x-2)^2} \\ &= \frac{(x-2)(3x^2) - (x^3 - 8)(1)}{(x-2)^2} \\ &= \frac{2x^3 - 6x^2 + 8}{(x-2)^2} \end{aligned}$$

(b)
$$\begin{aligned} f'(x) &= e^x \cdot \frac{\mathrm{d}}{\mathrm{dx}}(x^3) + x^3 \cdot \frac{\mathrm{d}}{\mathrm{dx}}(e^x) \\ &= e^x \cdot (3x^2) + x^3 \cdot e^x \\ &= e^x(x^3 + 3x^2) \end{aligned}$$

(c)
$$\begin{aligned} f'(x) &= \frac{\mathrm{d}}{\mathrm{dx}}(x^1) \\ &= 1 \end{aligned}$$

(d)
$$\begin{aligned} f'(x) &= \frac{e^x \cdot \frac{\mathrm{d}}{\mathrm{dx}}(x) - x \cdot \frac{\mathrm{d}}{\mathrm{dx}}(e^x)}{(e^x)^2} \\ &= \frac{e^x \cdot 1 - x \cdot e^x}{(e^x)^2} \\ &= \frac{e^x(1-x)}{(e^x)^2} \\ &= \frac{1-x}{e^x} \end{aligned}$$

Name _____

Quiz 5

1. Using the definition, find the equation of the tangent line of $f(x) = 1 - x^3$ at $x = -3$. The slope of the tangent line is

$$m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

where here $x = -3$. So

$$\begin{aligned} m_{tan} &= \lim_{h \to 0} \frac{f(-3+h) - f(-3)}{h} \\ &= \lim_{h \to 0} \frac{1 - (h-3)^3 - (1+27)}{h} \\ &= \lim_{h \to 0} \frac{-h^3 + 9h^2 - 27h}{h} \\ &= \lim_{h \to 0} (-h^2 + 9h - 27) \\ &= -27. \end{aligned}$$

The tangent line also goes through the point $(-3, f(-3)) = (-3, 28)$, so the equation (using point-slope form) is

$$y - 28 = -27(x+3).$$

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2. Find the derivatives of the given functions:

(a)
$$\begin{aligned} f'(x) &= \frac{e^x \frac{\mathrm{d}}{\mathrm{dx}}(x) + x \frac{\mathrm{d}}{\mathrm{dx}}(e^x)}{x^2} \\ &= \frac{e^x \cdot 1 + x \cdot e^x}{x^2} \\ &= \frac{e^x(x+1)}{x^2} \end{aligned}$$

(b)
$$\begin{aligned} f'(x) &= \frac{\mathrm{d}}{\mathrm{dx}}(x^{-1}) \\ &= -1 \cdot x^{-2} \end{aligned}$$

(c)
$$\begin{aligned} f'(x) &= x^2 \frac{\mathrm{d}}{\mathrm{dx}}(e^x) + e^x \frac{\mathrm{d}}{\mathrm{dx}}(x^2) \\ &= x^2 \cdot e^x + e^x \cdot 2x \\ &= e^x(x^2 + 2x) \end{aligned}$$

(d)
$$\begin{aligned} f'(x) &= \frac{(x+2)\,\frac{\mathrm{d}}{\mathrm{dx}}(x^3+8) + (x^3+8)\,\frac{\mathrm{d}}{\mathrm{dx}}(x+2)}{(x+2)^2} \\ &= \frac{(x+2)(3x^2) + (x^3+8)(1)}{(x+2)^2} \\ &= \frac{4x^3 + 6x^2 + 8}{(x+2)^2} \end{aligned}$$