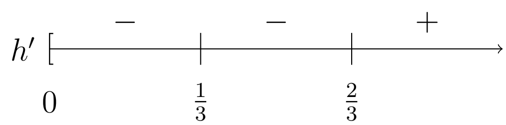
1. A balloon is released at time $t = 0$. The height of a balloon is given by

$$h(t) = 6 - t^2 + t^3,$$

where t is in seconds and h is in feet.

(a) (2 points) When is the balloon moving upwards?

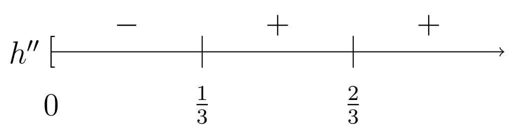
Solution. We find the times $t \geq 0$ such that $h'(t) > 0$. Observer $h'(t) = 3t^2 - 2t$ is a parabola that opens upwards and has zeros at $t = 0, 2/3$. Thus $h'(t)$ is positive for all $t > 2/3$. We exhibit the 'sign line' for $h'(t)$ below.



$$\boxed{(2/3, \infty)}$$

(b) (2 points) When is the balloon speeding up?

Solution. We find the times where $h''(t)$ has a different sign from $h'(t)$. Since we have the sign line for $h'(t)$ already, we look at $h''(t) = 6t - 2$. This is a line with positive slope and x-intercept $t = 1/3$, and thus is negative for $t < 1/3$ and is positive for $t > 1/3$.



Thus the only values of t for which the sign of $h'(t)$ differs from that of $h''(t)$ are values between $1/3$ and $2/3$.

$$\boxed{(1/3, 2/3)}$$

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2. The base of a right triangle at time t is given by $b(t) = t^2 + 1$. The height of the right triangle at time t is given by $h(t) = t^2 + t$. Find the rate at which the area of the triangle is changing at time t.

Solution. Since the area of a right triangle is equal to a half of the product of its base as its height, we have the area at time t is given by

$$\begin{aligned} A(t) &= \frac{1}{2}b(t)h(t) = \frac{1}{2}(t^2+1)(t^2+t) \\ &= \frac{1}{2}(t^4+t^3+t^2+t). \end{aligned}$$

Thus we have that its derivative is $A'(t) = \frac{1}{2}(4t^3 + 3t^2 + 2t + 1)$.

$$\boxed{\frac{1}{2}(4t^3+3t^2+2t+1)}$$