Name ______________

Quiz 6

1. Find the equation of the tangent line of $f(x) = \dfrac{\sin(x)}{x}$ at $x = \frac{\pi}{2}$.
First, we find the slope of the tangent line at $x = \frac{\pi}{2}$:

$$\begin{aligned} f'(x) &= \frac{x \frac{\mathrm{d}}{\mathrm{dx}}(\sin(x)) - \sin(x) \frac{\mathrm{d}}{\mathrm{dx}}(x)}{x^2} \\ &= \frac{x \cdot \cos(x) - \sin(x) \cdot 1}{x^2} \end{aligned}$$

So

$$\begin{aligned} f'(\frac{\pi}{2}) &= \frac{\frac{\pi}{2} \cdot \cos(\frac{\pi}{2}) - \sin(\frac{\pi}{2})}{(\frac{\pi}{2})^2} \\ &= -\frac{4}{\pi^2}. \end{aligned}$$

Our tangent line should pass through the point $(\frac{\pi}{2}, \frac{2}{\pi})$, so our line is

$$y - \frac{2}{\pi} = -\frac{4}{\pi^2}(x - \frac{\pi}{2}).$$

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2. Find the derivatives of the given functions:

(a) $f(x) = \dfrac{x^{1/2}}{\tan(x)}$

$$\begin{aligned} f'(x) &= \frac{\tan(x) \frac{\mathrm{d}}{\mathrm{dx}}(x^{1/2}) - x^{1/2} \frac{\mathrm{d}}{\mathrm{dx}}(\tan(x))}{\tan^2(x)} \\ &= \frac{\tan(x) \cdot \frac{1}{2}x^{-1/2} - x^{1/2} \sec^2(x)}{\tan^2(x)} \\ &= \frac{1}{2x^{1/2}} \cdot \cot(x) - x^{1/2} \csc^2(x) \end{aligned}$$

(b) $f(x) = \sec(x) \cdot x^2$

$$\begin{aligned} f'(x) &= x^2 \frac{\mathrm{d}}{\mathrm{dx}}(\sec(x)) + \sec(x) \frac{\mathrm{d}}{\mathrm{dx}}(x^2) \\ &= x^2 \sec(x)\tan(x) + \sec(x) \cdot 2x \end{aligned}$$

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3. Find the second derivative of $f(x) = e^2 \cdot \cos(x)$.

$$\begin{aligned} f'(x) &= e^2 \frac{\mathrm{d}}{\mathrm{dx}}(\cos(x)) \\ &= e^2(-\sin(x)) \\ f''(x) &= \frac{\mathrm{d}}{\mathrm{dx}} f'(x) \\ &= -e^2 \frac{\mathrm{d}}{\mathrm{dx}}(\sin(x)) \\ &= -e^2 \cdot \cos(x) \end{aligned}$$

Name ______________

Quiz 6

1. Find the equation of the tangent line of $f(x) = \dfrac{\cos(x)}{x}$ at $x = \pi$.

First, we find the slope of the tangent line at $x = \pi$:

$$\begin{aligned} f'(x) &= \frac{x \frac{\mathrm{d}}{\mathrm{dx}}(\cos(x)) - \cos(x) \frac{\mathrm{d}}{\mathrm{dx}}(x)}{x^2} \\ &= \frac{x(-\sin(x)) - \cos(x) \cdot 1}{x^2} \end{aligned}$$

So

$$\begin{aligned} f'(\pi) &= \frac{\pi \cdot (-\sin(\pi)) - \cos(\pi)}{\pi^2} \\ &= \frac{1}{\pi^2} \end{aligned}$$

Our tangent line should pass through the point $(\pi, -\frac{1}{\pi})$, so our line is

$$y + \frac{1}{\pi} = \frac{1}{\pi^2}(x - \pi).$$

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2. Find the derivatives of the given functions:

(a) $f(x) = x^2 \cdot \csc(x)$

$$\begin{aligned} f'(x) &= x^2 \frac{\mathrm{d}}{\mathrm{dx}}(\csc(x)) + \csc(x) \frac{\mathrm{d}}{\mathrm{dx}}(x^2) \\ &= x^2(-\csc(x)\cot(x)) + \csc(x)(2x) \end{aligned}$$

(b) $f(x) = \dfrac{x^{-1/2}}{\cot(x)}$

$$\begin{aligned} f'(x) &= \frac{\cot(x) \frac{\mathrm{d}}{\mathrm{dx}}(x^{-1/2}) - x^{-1/2} \frac{\mathrm{d}}{\mathrm{dx}}(\cot(x))}{\cot^2(x)} \\ &= \frac{\cot(x)(-\frac{1}{2}x^{-3/2}) - x^{-1/2}(-\csc^2(x))}{\cot^2(x)} \\ &= -\frac{1}{2x^{3/2}} \cdot \tan(x) + \frac{1}{x^{1/2}} \sec^2(x) \end{aligned}$$

3. Find the second derivative of $f(x) = e^x \cdot \sin(x)$.

$$\begin{aligned}
f'(x) &= e^x \frac{\mathrm{d}}{\mathrm{dx}}(\sin(x)) + \sin(x) \frac{\mathrm{d}}{\mathrm{dx}}(e^x) \\
&= e^x(\cos(x)) + \sin(x)e^x \\
&= e^x(\cos(x) + \sin(x)) \\
f''(x) &= e^x \frac{\mathrm{d}}{\mathrm{dx}}(\cos(x) + \sin(x)) + (\cos(x) + \sin(x)) \frac{\mathrm{d}}{\mathrm{dx}} e^x \\
&= e^x(-\sin(x) + \cos(x)) + (\cos(x) + \sin(x))e^x \\
&= -e^x \sin(x) + e^x \cos(x) + e^x \cos(x) + e^x \sin(x) \\
&= 2e^x \cos(x)
\end{aligned}$$